Control of Partial Differential Equations In Hauts-De-France (1^{st} edition)

06-08 November, 2023

Université Polytechnique Hauts-de-France (Valenciennes)

Laboratoire de Matériaux Céramiques et de Mathématiques - CERAMATHS

Organised by

Mohammad AKIL(CERAMATHS)-Valenciennes.
Olivier Goubet (Laboratoire Paul Painlevé)-Lille.
Serge Nicaise (CERAMATHS)-Valenciennes.
Lionel Rosier (LMPA)- Calais.

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- 11:00-12:00: **Emmanuelle Crépeau**: *Stabilization results for KdV equations with delay.*
- 12:00-13:30: **Lunch Break**.
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1.2 Tuesday 07/11

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1.3 Wednesday 08/11

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- 10h00-11h00: **Julie Valein**: *Carleman-based reconstruction algorithm on a wave network.*
- 11h00-11h30: **Coffee Break**.
- 11h30-12h00: **Damien Galant**: *Study of the effect of parameters on the decay rate of a fourth order problem.*
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- 12h30-14h00: **Lunch Break**.
- 14h00-15h00: **Gerd Wachsmuth**: *Optimal control and the obstacle problem.*
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2 Abstracts

2.1 Abstracts-Monday 06/11

. • **10h00-11h00**.

A hierarchy of heat conduction laws
Vittorino Pata

We produce a family of equations describing the evolution of the temperature in a rigid heat conductor. This is obtained by means of successive approximations of the Fourier law, via memory relaxations and integral perturbations. This talk is based on a joint paper with Filippo Dell'Oro.

• **11h00-12h00**.

Stabilization results for KdV equations with delay
Emmanuelle Crépeau

In this talk, the exponential stability of the nonlinear Korteweg-de Vries equation with delayed terms is considered both in the case of a bounded interval and a tree-shape network. We will give two types of proof, one constructive with some Lyapunov techniques and the other more general with observability results. We will also discuss the results from a numerical point of view. This is joint work with L. Baudouin, J. Valein, C. Prieur and H. Parada.

• **12h00-13h30**. **Lunch Break.**

• **13h30-14h30**.

Energy decay estimates for semilinear wave-type equations with time-dependent time delay

Cristina Pignotti

We analyze a class of semilinear damped wave-type equations with a delay feedback with time-variable time delay. By combining semigroup arguments, careful energy estimates, and an iterative approach we are able to prove, under suitable assumptions, a well-posedness result, and an exponential decay estimate for solutions corresponding to small initial data.

• **14h30-15h30**.

Recent Progresses and Open Problems in Controlling Magnetizable Piezoelectric Systems

Ahmet Özkan Özer

Magnetizable piezoelectric materials exhibit electromagnetic responses to mechanical stress, and mechanical responses to electromagnetic stress. Therefore, they are actively used as actuators and sensors in smart-material system designs. In this talk, several PDE models for various piezoelectric systems are proposed. These models mainly couple the longitudinal and/or transverse vibrations of layers with the electro-magnetism due to Maxwell's equations and/or thermal effects. Recent results on the exact boundary observability, controllability, stabilizability with the inclusion of state feedback, delay, long-range memory, nonlinearity, partial viscous/fractional damping, and observer design will be discussed. Certain mathematical hurdles will be mentioned. More importantly, the approximations of these PDE models by the standard Finite-Elements and Finite-Differences suffer from mimicking control-theoretic behaviors uniformly with respect to the discretization parameter. Recently proposed remedies, involving the order-reduction-based algorithms, will be discussed since they provide robust approximations. Finally, if the time allows, ongoing and recently published Wolfram's Demonstration Projects, which are interactive visualizations with realistic material parameters. This platform allows the controllers to be adjusted in real-time. Related future directions will be discussed.

• **15h30-16h00**. **Coffee Break.**

• **16h00-17h00**.

Exponential stability of the linear KdV-BBM equation with point dissipation

Zhuangyi Liu

In this talk, we consider the linear Korteweg-de-Vries-Benjamin Bona Mahony (Kdv-BBM) equation on a finite interval $[0, L]$ with point dissipation boundary conditions. We show the well-posedness by the semigroup theory. A set of critical length L is obtained, for which the system possesses conservative solutions. Then we investigate the stability of the associated semigroup for the case of non-critical length by the frequency domain method.

• **17h00-17h30**.

Exponential synchronization of Kumamoto oscillators with time-delayed interactions

Chiara Cicolani

We study the asymptotic synchronization for the Kuramoto oscillators model with time-delayed interactions. The Kuramoto model appears in many biological/physiological applications. Then, often, time delay effects have to be considered. We provide an explicit lower bound on the coupling strength and an upper bound on the time delay in terms of initial configurations ensuring exponential synchronization. Our approach, which relies on continuity arguments and careful estimates of the trajectories, allows us to significantly relax previous thresholds on the time delay size. We are able to consider both the cases with initial phase diameter less or greater than $\frac{\pi}{2}$.

• **17h30-18h00**.

Stability for degenerate wave equations with drift under simultaneous degenerate damping

Ibtissam Issa

In this talk, we study the stability of two different problems. The first problem is a one-dimensional degenerate wave equation with degenerate damping, incorporating a drift term and a leading operator in non-divergence form. In the second problem, we consider a system that couples degenerate and non-degenerate wave equations, connected through transmission, and subject to a single dissipation law at the boundary of the non-degenerate equation. In both scenarios, we derive exponential stability results. This is joint work with Mohammad Akil and Genni Fragnelli.

• **18h00-18h30**.

Indirect Stabilization of Locally Weakly Coupled Second Order Evolution Systems of Hyperbolic Type

Farah Trad

The purpose of our work is to investigate the stabilization of locally weakly coupled second order evolution equations of hyperbolic type, where only one of the two equations is directly damped. As such system cannot be exponentially stable we are interested in polynomial energy decay rates. Our main contributions concern abstract polynomial stability properties based on stability properties of two auxilliary problems. Our abstract framework is illustrated by several concrete examples not treated before.

2.2 Abstracts-Tuesday 07/11

. • **09h00-10h00**.

From microscopic to macroscopic scale equations: mean field, hydrodynamic and graph limits

Emmanuel Trélat

Considering finite particle systems, we elaborate on various ways to pass to the limit as the number of agents tends to infinity, either by mean field limit, deriving the Vlasov equation, or by hydrodynamic or graph limit, obtaining the Euler equation. We provide convergence estimates. We also show how to pass from Liouville to Vlasov or to Euler by taking adequate moments. Our results encompass and generalize a number of known results of the literature. As a surprising consequence of our

analysis, we show that sufficiently regular solutions of any quasilinear PDE can be approximated by solutions of systems of N particles, to within 1/log(log(N)). This is a work with Thierry Paul.

• **10h00-11h00**.

A theoretical approach to smart and composite materials
Genni Fragnelli

We will discuss a very recent mathematical approach to the study of composite and smart materials. In spite of the huge amount of numerical results, a rigorous theoretical approach is still missing. After introducing the needed mathematical setting for these classes of problems, we will provide some existence results and the description of the behaviour of the solutions for some concrete models.

• **11h00-11h30**. **Coffee Break**. • **11h30-12h00**.

The Cucker-Smale model with attractive repulsive interaction
Elisa Continelli

We consider a minimum time problem with dynamics depending on the past history of the trajctories. As a consequence, the value function of the investigated problem, called the minimum time function, its no longer defined in a subset $\mathbb{R}^n$ of the Banach space of all continuous functions from $[-\tau, 0]$ to $\mathbb{R}^n$. The Hamilto-Jacobi theory for optima control problems with delay has been developed by several authors. Our analysis is instead about the regularity of the minimum time functional. Extending some classical results, we prove the semiconcavity of the minimum time functional in a suitable subset of the reachable set. Join work with C. Pignotti.

• **12h00-12h30**.

A new approach on the controllability of the nonlinear Fokker-Planck equations
Gastón Vergara Hermosila

In this talk we consider some aspects about the controllability of the nonlinear Fokker-Planck equation on a bounded domain of $\mathbb{R}^n$. The system is completed with Robin boundary conditions. We will show the existence of a feedback control which satisfies an appropriate elliptic equation. In a second part we will compare our main theorem with the results in the literature of control and transport optimal. Finally we will present some ideas about the extension of our methods to unbounded domains.

• **12h30-14h00**. **Lunch Break.**

• **14h00-15h00**.

Lie brackets and interpolation for controllability
Karine Beauchard

This talk will survey old and recent results on the local controllability of control systems modeled by ODEs, focussing on results stated using Lie brackets of the vector fields defining the dynamics. We will propose a unified approach to determine and prove obstructions to local controllability. This approach relies on a recent Magnus-type representation formula of the state, a new Hall basis of the free Lie algebra over two generators and Gagliardo-Nirenberg interpolation inequalities. This approach allows to recover the known necessary conditions, but also to prove a conjecture of 1986 due to Kawski and many other new necessary conditions. Finally, we will see how these results

translate for PDEs, in particular the Schrödinger equation. This is a joint work with Frederic Marbach and Jeremy Le Borgne.

• **15h00-16h00**.

On the stability of wave-heat systems with Coleman-Gurtin thermal law
Filippo Dell'Oro

We study the long-term behaviour of solutions to a one-dimensional coupled wave-heat system with Coleman-Gurtin thermal law. Our approach is based on the asymptotic theory of C_0-semigroups and recent results developed for coupled control systems. As our main results, we represent the system as a feedback interconnection between the wave part and the Coleman-Gurtin part and we show that the associated semigroup in the history framework of Dafermos is polynomially stable with optimal decay rate t^{-2}. • **16h00-16h30**. **Coffee Break**.

• **16h30-17h00**.

Boundary null controllability of convection-diffusion equations with constraints on the state and application to the identification of boundary pollution parameters
Arnaud Fournier

We study a null controllability problem for a convection-diffusion with the control contained in Fourrier boundary condition to identify M parameters of pollution. The results are achieved by means of an observability inequality derived from new Carleman estimates for the boundary null controllability and the construction of a boundary sentinel to identify the parameters.

• **17h00-17h30**.

Optimal control of the temperature by the laser path, the laser power and the thermal treatment time in selective laser melting process
Hiba Hmede

Additive manufacturing by laser fusion on metal oxides powder bed such as e.g. Aluminia (Al_2O_3) or aluminium titanate (Al_2TiO_5) has developed considerably in the last few years and allows today the production of a wide range of complex parts. The mathematical problem considered in this talk, is to control the distribution of temperature inside some part of the powder layer, distribution of temperature solution of an initial boundary value problem with a heat source corresponding to the laser trajectory at some variable power on the upper surface of the powder layer. The main questions concern the optimization of the trajectories of the laser and of its power according to given criteria: imposing that during the melting process the temperature reachs a melting value in the structure to be built, minimize the thermal gradients, all this in the shortest thermal treatment time possible. An appropriate cost functional is introduced, first order necessary optimal conditions are derived and the C^2 Fréchet differentiability of the reduced cost functional is also studied. This material is part of our doctoral thesis.

• **17h30-18h30**.

Optimal placement of sensors and actuators
Enrique Zuazua

In this lecture we shall discuss the problem of sensor and actuator placement in control. First of all, we shall present our earlier work in collaboration with Y. Privat and E. Trélat based in Fourier analysis. We shall later introduce the more recent work in collaboration with I. Ftouhi in which the

problem is addressed from a purely geometric perspective. We also present some challenging open problems.

• **19h30**. **Gala Dinner**.

2.3 Abstracts-Wednesday 08/11

. • **09h00-10h00**.

Nonhomogeneous Boundary Value Problems of a Class of Nonlinear Dispersive Wave Equations

Bing-yu Zhang

In this presentation, we will delve into the initial boundary value problems (IBVP) associated with a specific class of nonlinear dispersive wave equations defined on a finite interval $(0, L)$. These equations take the form:

$$\begin{cases} \partial_t u + \sum_{j=0}^{2n} a_j \partial_x^j u + \partial_x^{2n+1} u = N(u, \partial_x u, \dots, \partial_x^{2n-1} u), \quad \text{for } x \in (0, L),\ t > 0, \\ \\ u(x, 0) = \phi(x), \end{cases} \tag{1}$$

where we impose nonhomogeneous boundary conditions as follows:

$$\mathcal{B}_k u = h_k(t), \quad \text{for } k = 1, 2, \dots, 2n+1, \tag{2}$$

with

$$\mathcal{B}_k u := \sum_{j=0}^{2n} \alpha_{k,j} \partial_x^j u(0, t) + \beta_{k,j} \partial_x^j u(L, t) = h_k$$

and the coefficients $\alpha_{k,j}, \beta_{k,j}$ are real constants, for $k = 1, 2, \cdots, 2n+1; j = 0, 1, \cdots, 2n$.

Our primary objective in this presentation is to address the following question:

Under what conditions on the coefficients α_{kj} and β_{kj} does the IBVP become well-posed in the classical Sobolev space $H^s(0, L)$, with initial data ϕ belonging to $H^s(0, L)$ and boundary values h_k belonging to some appropriate spaces with optimal regularity?

By clarifying the conditions for well-posedness, our aim is to provide valuable insights into the study of boundary controllability and stabilizability for these nonlinear dispersive wave equations.

• **10h00-11h00**.

Carleman-based reconstruction algorithm on a wave network

Julie Valein

In this article, we are interested in an inverse problem set on a tree shaped network where each edge behaves according to the wave equation with potential, external nodes have Dirichlet boundary conditions and internal nodes follow the Kirchoff law. The main goal is the reconstruction of the potential everywhere on the network, from the Neumann boundary measurements at all but one external vertices. Leveraging from the Lipschitz stability of this inverse problem, we aim at providing an efficient reconstruction algorithm based on the use of an appropriate global Carleman estimate. This is a joint work with Lucie Baudouin, Maya de Buhan and Emmanuelle Crépeau.

• **11h00-11h30**. **Coffee Break**.

• **11h30-12h00**.

Carleman-based reconstruction algorithm on a wave network

Damian Galant

We consider the fourth-order evolution problem

$$\begin{cases} \partial_t^2 u(x,t) + a\partial_x^4 u(x,t) + b\partial_t u(x,t) + (\alpha\partial_t u(\xi,t) + \beta u(\xi,t))\,\delta_\xi = 0, & (x,t) \in (0,1)\times(0,+\infty), \\ u(0,t) = u(1,t) = \partial_x^2 u(0,t) = \partial_x^2 u(1,t) = 0 & t \in (0,+\infty), \\ u(x,0) = u_0(x), \quad \partial_t u(x,0) = u_1(x) & x \in (0,1), \end{cases} \tag{3}$$

where a, b, α and β are positive constants and δ_ξ is the Dirac mass at the point $\xi \in (0;1)$. This problem is motivated by the study of Shape Memory Alloy cables in bridges (see [1]).

Taking $U := (u, \partial_t u)$, the problem can be formally written as the evolution equation $\partial_t U = \mathcal{A}U$, where the unbounded operator $\mathcal{A}$ is defined by

$$\mathcal{A}(u,v) := \big(v, -a\partial_{xxxx}u - bv - [\alpha v(\xi) + \beta u(\xi)]\delta_\xi\big)$$

on the domain

$$D(\mathcal{A}) := \Big\{ (u,v) \in \big[H^4(0,\xi)\cap H^4(\xi,1)\cap H^2(0;1)\big]\times H^2(0;1),\ u(0)=v(0)=u(1)=v(1)=0,$$
$$\partial_{xx}u(0) = \partial_{xx}u(1) = 0,\ \partial_{xx}u(\xi^-) = \partial_{xx}u(\xi^+),\ \partial_{xxx}u(\xi^+) - \partial_{xxx}u(\xi^-) = -\frac{\alpha}{a}v(\xi) - \frac{\beta}{a}u(\xi)\Big\}.$$

In [2][1], a spectral approach was used to study the impact of α on the decay rate of solutions to problem (3). In particular, the fastest decay rate of problem (3) is given by the supremum of the real part of the spectrum of $\mathcal{A}$ since the existence of a Riesz basis is proved. While all eigenvalues of $\mathcal{A}$ can be explicitly computed when $\alpha = \beta = 0$, there is no hope to do so for $\alpha \neq 0$ due to the complicated form of the characteristic equation.

In this talk, I will present how to study qualitatively the dependence of the optimal decay rate on α and β. The main tools of this study are continuation arguments and the holomorphic dependence of eigenvalues on the parameters coupled with some careful "a priori estimates" on the eigenvalues.

This study shows the (perhaps surprising) fact that the decay rate of the problem gets *worse* as α increases, showing that the term "$\alpha\partial_t u(\xi,t)\delta_\xi$" does *not* act as a damping term in the problem.

This is joint work with Virginie Régnier (CERAMATHS/DMATHS, UPHF) and Christophe Troestler (Département de Mathématique, UMONS).

References

[1] A-R. Liu, C-H. Liu, J-Y. Fu, Y-L. Pi, Y-H. Huang, J-P. Zhang, A Method of Reinforcement and Vibration Reduction of Girder Bridges Using Shape Memory Alloy Cables, Int. J. Struct. Stab. Dyn. **17** (No. 7) (2017) 1750076.

[2] V. Régnier. Do Shape Memory Alloy cables restrain the vibrations of girder bridges? - A mathematical point of view. ESAIM: COCV, vol. 29, No 16 (2023), 24 pages.

[1] Work presented by Virginie Régnier at the 2022 edition of "A day of control on PDEs".

• **12h00-12h30**.

General energy decay result for a viscoelastic swelling porous-elastic system

Abderrahmane Youkana

In this work, we consider the following viscoelastic swelling porous-elastic system

$$\begin{cases} \rho_z z_{tt} - a_1 z_{xx} - a_2 u_{xx} + \int_0^t g(t-s) z_{xx}(x,s)\, ds = 0, & \text{in } (0,1)\times(0,\infty), \\ \rho_u u_{tt} - a_3 u_{xx} - a_2 z_{xx} = 0, & \text{in } (0,1)\times(0,\infty), \end{cases} \tag{4}$$

and establish explicit energy decay result with a very general assumption on the behavior of g at infinity; that is

$$g'(t) \leq -k(t) H\left(g(t)\right), \quad t \geq 0, \tag{5}$$

where k and H are two functions satisfying some conditions mentioned in the paper. Our result substantially improves earlier related result in the literature.

• **12h30-14h00**. **Lunch Break**.

• **14h00-15h00**.

Optimal control and the obstacle problem

Gerd Wachsmuth

This talk is about optimal control problems for which an obstacle problem comes into play. On the one hand, we speak about the optimal control of the obstacle problem (in particular optimality conditions of first and second order). On the other hand, we consider an optimal control problem with H^1-regularization and bound constraints. For this problem, the first-order optimality system contains an obstacle problem and we demonstrate that this system can be solved by means of a semismooth Newton method.

• **15h00-15h30**.

Optimal control of a class of semilinear fractional elliptic equations

Steve-Cyrille Kenne

In this work, a class of semilinear fractional elliptic equations associated to the spectral fractional Dirichlet Laplace operator is considered. After showing several existence and regularity results, we establish the existence of optimal solutions as well as a minimum principle of Pontryagin type and the first order necessary optimality conditions of associated optimal control problems. Second order conditions for optimality are also obtained for L^∞ and L^2-local solution under some structural assumptions.

• **15h30-16h00**. **Coffee Break**.

• **16h00-17h00**.

Stability of distributed systems involving wave and Kirchhoff plate equations with dynamical boundary controls

Ali Wehbe

We investigate the energy decay of distributed systems involving wave and Kirchhoff plate equations with dynamical boundary controls. First, by spectrum approach combining with a general criteria of Arendt-Batty, we prove that our model is strongly stable. Then, after proving that this system lacks the exponential stability, we establish different type of polynomial energy decay rates. Finally, we prove that the obtained energy decay rate is optimal in particular case.