BONUS EXERCISE PART 1: PRACTICAL COURSE MODELING, SIMULATION, OPTIMIZATION

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We consider the transversal displacement $w(t, x, y)$ in the aluminum plate of $L_x = 0.4$ [m] by $L_y = 0.3$ [m] with a hole shown in Figure 1. We use a Kirchoff-Love plate model

$$\rho H \frac{\partial^2 w}{\partial t^2} + \frac{EH^3}{12(1-\nu^2)} \nabla^4 w = 0,$$

where $\rho = 2700$ [kg/m^3] is the mass density, $H = 0.01$ [m] is the thickness, $E = 69 \cdot 10^9$ [Pa = N/m^2] is the Young's modulus, $\nu = 0.3$ [-] is Poisson's ratio, and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

We do not apply forces or moments on the boundary. The weak form of this problem then takes the form

$$\int_0^{L_x} \int_0^{L_y} \rho H v \frac{\partial^2 w}{\partial t^2} \, \mathrm{d}y \, \mathrm{d}x +$$

$$\int_0^{L_x} \int_0^{L_y} \begin{bmatrix} \frac{\partial^2 v}{\partial x^2} & \frac{\partial^2 v}{\partial y^2} & 2\frac{\partial^2 v}{\partial x \partial y} \end{bmatrix} \frac{EH^3}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2\frac{\partial^2 w}{\partial x \partial y} \end{bmatrix} \mathrm{d}y \, \mathrm{d}x = 0.$$

where v is a test function in $H^2([0, L_x] \times [0, L_y])$.

Compute the lowest 10 eigenfrequencies and plot the corresponding eigenmodes (= eigenfunctions). Indicate whether the unit of the eigenfrequencies you computed is rad/s or Hz.

Hint: The weak formulation of this problem now requires shape functions to be in H^2, which means that the FE shape functions should be at least C^1. We can construct C^1 FE shape functions using the following 4 Hermite polynomials

$$p_0^w(\xi) = 1 - 3\xi^2 + 2\xi^3, \qquad p_1^w(\xi) = 3\xi^2 - 2\xi^3,$$
$$p_0^\theta(\xi) = \xi - 2\xi^2 + \xi^3, \qquad p_1^\theta(\xi) = -\xi^2 + \xi^3, \qquad \xi \in [0, 1].$$

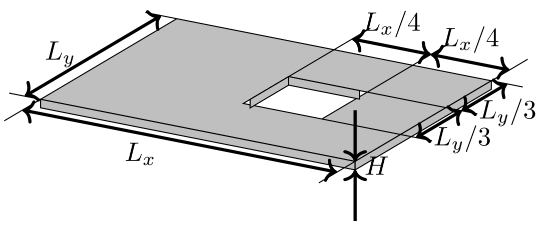

FIGURE 1. The considered aluminum plate

To understand these definitions, check the values $p(\xi)$ and $\frac{\mathrm{d}p}{\mathrm{d}\xi}(\xi)$ for $\xi \in \{0,1\}$.
The shape function in a rectangular $L_{e,x} \times L_{e,y}$-element can then be chosen as $\mathbf{N}^e(x/L_{e,x}, y/L_{e,y})$, where

$$\mathbf{N}^e(\xi,\eta) = \begin{bmatrix} p_0^w(\xi)p_0^w(\eta) \\ p_1^w(\xi)p_0^w(\eta) \\ p_1^w(\xi)p_1^w(\eta) \\ p_0^w(\xi)p_1^w(\eta) \\ p_0^\theta(\xi)p_0^w(\eta)L_{e,x} \\ p_1^\theta(\xi)p_0^w(\eta)L_{e,x} \\ p_1^\theta(\xi)p_1^w(\eta)L_{e,x} \\ p_0^\theta(\xi)p_1^w(\eta)L_{e,x} \\ p_0^w(\xi)p_0^\theta(\eta)L_{e,y} \\ p_1^w(\xi)p_0^\theta(\eta)L_{e,y} \\ p_1^w(\xi)p_1^\theta(\eta)L_{e,y} \\ p_0^w(\xi)p_1^\theta(\eta)L_{e,y} \\ p_0^\theta(\xi)p_0^\theta(\eta)L_{e,x}L_{e,y} \\ p_1^\theta(\xi)p_0^\theta(\eta)L_{e,x}L_{e,y} \\ p_1^\theta(\xi)p_1^\theta(\eta)L_{e,x}L_{e,y} \\ p_0^\theta(\xi)p_1^\theta(\eta)L_{e,x}L_{e,y} \end{bmatrix}^\top .$$

Note that there are four degrees of freedom per node: w, $\frac{\partial w}{\partial x}$, $\frac{\partial w}{\partial y}$, and $\frac{\partial^2 w}{\partial x \partial y}$.