EXERCISE WEEK 2:
PRACTICAL COURSE
MODELING, SIMULATION, OPTIMIZATION

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We consider heat conduction in the copper plate in Figure 1. The plate is $L_x = 0.5$ [m] by $L_y = 0.75$ [m] and has a thickness of $H = 0.01$ [m] and a thermal conductivity of $k = 400$ [W/m/K]. Because the plate is thin, the steady state temperature increase T only depends on the in-plane coordinates x and y, i.e. $T(x, y)$. The plate is subjected to a heat load

$$Q(x, y) = Q_0 \exp\left(-\frac{(x - x_0)^2 + (y - y_0)^2}{a^2}\right),$$

where the center of the heat load is at $(x_0, y_0) = 0.3(L_x, L_y)$, the width parameter is $a = 0.05$ [m], and the intensity is $Q_0 = 100$ [W/m^2].

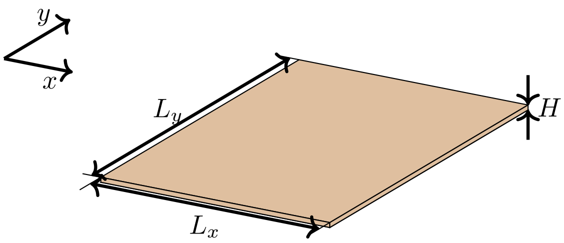

FIGURE 1. The considered copper plate

The boundary conditions at the four edges are given by

$$H\mathbf{q} \cdot \mathbf{n} = hT,$$

where the heat flux $\mathbf{q} = [q_x, q_y]^\top$ is given by Fourier's law of heat conduction, $\mathbf{n} = [n_x, n_y]^\top$ is the outward pointing unit normal to the edge, and the thermal conductance between the boundary and the environment is $h = 5$ [W/m/K].

a. (3pts) Write down the partial differential equation for the steady-state temperature increase in the plate $T(x, y)$. Write the boundary conditions for all four edges separately without using the unit normal $\mathbf{n}$.

b. (5pts) Discretize the problem formulated at a. by finite differences. Use $N_x = 32$ grid points in the x-direction and $N_y = 42$ grid points in the y-direction.
Hint: You can use the implicit or the explicit formulation for the boundary conditions. Depending on your choice, you can start from the files `Week2_exercise_explicit` or `Week2_exercise_implicit`.

c. (2pts) What happens when you set the thermal conductance $h = 0$, i.e. when you use zero Neumann boundary conditions on all four edges? Can you explain the problem by looking at the PDE formulated at a.?