EXERCISE WEEK 6:
PRACTICAL COURSE
MODELING, SIMULATION, OPTIMIZATION

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We again consider the steady-state temperature distribution in the copper plate in Figure 1 that we also considered in Exercise 2. The plate is $L_x = 0.5$ [m] by $L_y = 0.75$ [m], has a thickness of $H = 0.01$ [m], and a thermal conductivity of $k = 400$ [W/m/K]. Because the plate is thin, the steady state temperature increase T only depends on the in-plane coordinates x and y, i.e. $T(x, y)$. The plate is subjected to a heat load

$$Q(x, y) = Q_0 \exp\left(-\frac{(x - x_0)^2 + (y - y_0)^2}{a^2}\right),$$

where the center of the heat load is at $(x_0, y_0) = 0.3(L_x, L_y)$, the width parameter is $a = 0.05$ [m], and the intensity is $Q_0 = 100$ [W/m^2].

The boundary conditions at the four edges are given by

$$H\mathbf{q} \cdot \mathbf{n} = hT,$$

where the heat flux $\mathbf{q} = [q_x, q_y]^\top$ is given by Fourier's law of heat conduction, $\mathbf{n} = [n_x, n_y]^\top$ is the outward pointing unit normal to the edge, and the thermal conductance between the boundary and the environment is $h = 5$ [W/m/K].

a. (3pts) Write the weak form of the boundary value problem for the steady-state temperature increase $T(x, y)$.
b. (5pts) Discretize the problem formulated at a. using linear rectangular finite elements. Use $M_x = 32$ elements in the x-direction and $M_y = 42$ elements in the y-direction. To compute the load vector, you can assume that $Q(x, y)$ is constant inside each element.
Hint: Follow the approach from `Week6_lecture.pdf`.
Hint: You can use the file `Week6_exerciseb` a starting point.
c. (2pts) Modify the code from part b. in (at least) one of the following ways.
   i. Replace the linear shape functions inside each element by quadratic shape functions.

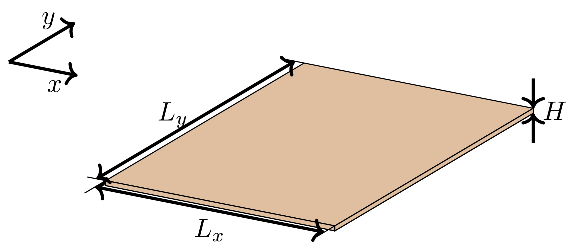

FIGURE 1. The considered copper plate

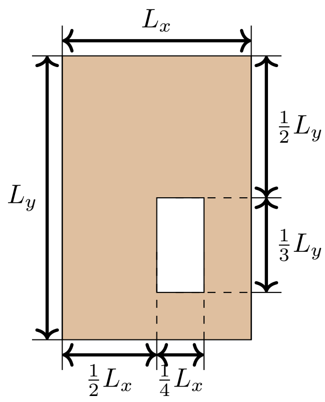

FIGURE 2. The location of the hole in the plate

Hint: The numbering of the nodes is much easier when you use the quadratic shape functions that have 9 nodes inside each element instead of the serendipity elements with 8 nodes inside each element.

ii. Create a hole in the plate at the location indicated in Figure 2. For simplicity, you can consider zero Neumann boundary conditions on the boundary of the hole (the boundary conditions on the other edges do not change).
Hint: Note that the boundaries of the hole coincide with the element boundaries in the considered mesh.
Hint: In the most elegant solution, you do not assign node numbers to nodes inside the hole.