EXERCISE WEEK 8:
PRACTICAL COURSE
MODELING, SIMULATION, OPTIMIZATION

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We consider the aluminum beam in Figure 1 of length $L = 0.4$ [m]. The beam has a mass density $\rho = 2700$ [kg/m^3] and a Young's modulus $E = 69$ GPa. The beam has a rectangular cross section of $W = 0.02$ [m] by $H = 0.01$ [m]. We model the beam as an Euler-Bernoulli beam.

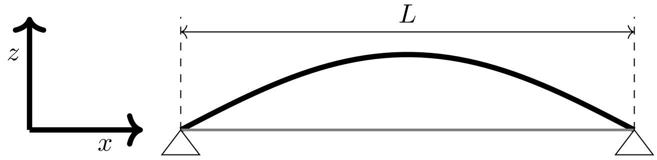

FIGURE 1. The considered pinned-pinned beam

a. (5pts) Make a FE discretization of the considered beam with $M = 100$ elements. Compute the six first (lowest) angular eigenfrequencies ω_k [rad/s] for this FE model. Plot the corresponding eigenmodes $\mathbf{w}_k$.
Hint: You can use the file `Week8_exercisea` as a starting point.
Hint: for every eigenmode $\mathbf{w}_k$, the discretized model has a solution $\mathbf{w}(t) = \sin(\omega_k t)\mathbf{w}_k$.
Hint: It is most efficient to use `eigs(A,B,6,'sm')` to find the six smallest eigenvalues of the generalized eigenvalue problem $(A - \lambda B)v = 0$.

The eigenmodes $w_k(x)$ and eigenfrequencies ω_k (in rad/s) of the considered beam model can also be determined analytically as

$$w_k(x) = \sin(\tfrac{k\pi x}{L}), \qquad \omega_k = \sqrt{\frac{EI}{\rho A}}\left(\frac{k\pi}{L}\right)^2.$$

b. (3pt) Compare the eigenvalues in the FE models with $M = 10$, 30, 100, 300, 1000, 3000, and 10,000 elements to the analytic values. Visualize your results in a plot. What do you conclude?
Hint: You can use the file `Week8_exerciseb` as a starting point.

c. (3pts) Answer one of the following questions:
   i. Explain why the first eigenfrequency of the FE model is always higher than the first analytical eigenfrequency.
   Hint: Consider the Rayleigh quotient.
   ii. Consider the aluminum cantilever beam of length $L = 0.4$ [m] and with rectangular cross section $W = 0.02$ [m] by $H = 0.01$ [m] in Figure 2. A damper with damping constant $b = 5$ [Ns/m] is attached at the free end, meaning that $Q(L) = -b\dot{w}(t, L)$. Compute the first twelve eigenvalues λ_k for this beam model. Compare the obtained 12 eigenvalues λ_k to the first six undamped (so with $b = 0$) angular

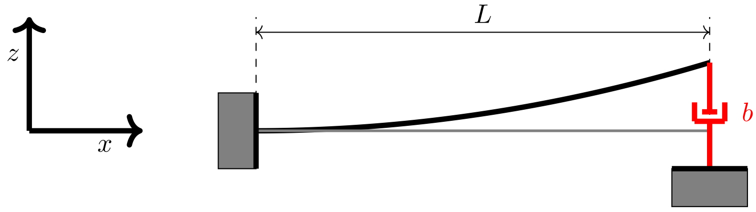

FIGURE 2. The considered damped cantilever beam

eigenfrequencies ω_k of the considered cantilever beam. What do you observe?
Hint: For a cantilever beam, the displacement and rotation on one end are fixed and the other end is free, i.e. no forces and moments are applied.
Hint: The discretized system can be written in the form

$$\mathbf{M}\ddot{\mathbf{w}}(t) + \mathbf{B}\dot{\mathbf{w}}(t) + \mathbf{K}\mathbf{w}(t) = \mathbf{f}(t),$$

where $\mathbf{B}$ is the damping matrix. We can rewrite these equations are

$$\begin{bmatrix} \mathbf{M} & 0 \\ 0 & \mathbf{M} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{w}}(t) \\ \ddot{\mathbf{w}}(t) \end{bmatrix} + \begin{bmatrix} 0 & -\mathbf{M} \\ \mathbf{K} & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{w}(t) \\ \dot{\mathbf{w}}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{f}(t) \end{bmatrix}.$$

Hint: an eigenvalues λ_k leads to a solution $\mathbf{w}(t) = e^{\lambda_k t}\mathbf{w}_k$, where $\mathbf{w}_k$ is the corresponding eigenmode.
Extra: You now do not need to plot the corresponding eigenmodes $\mathbf{w}_k$. What is the problem you encounter when you try to?