FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG
FACULTY OF SCIENCES

Chair DYNAMICS, CONTROL AND NUMERICS FAU

THE SHEEP HERDING GAME: CAN YOU BEAT THE ALGORITHM?

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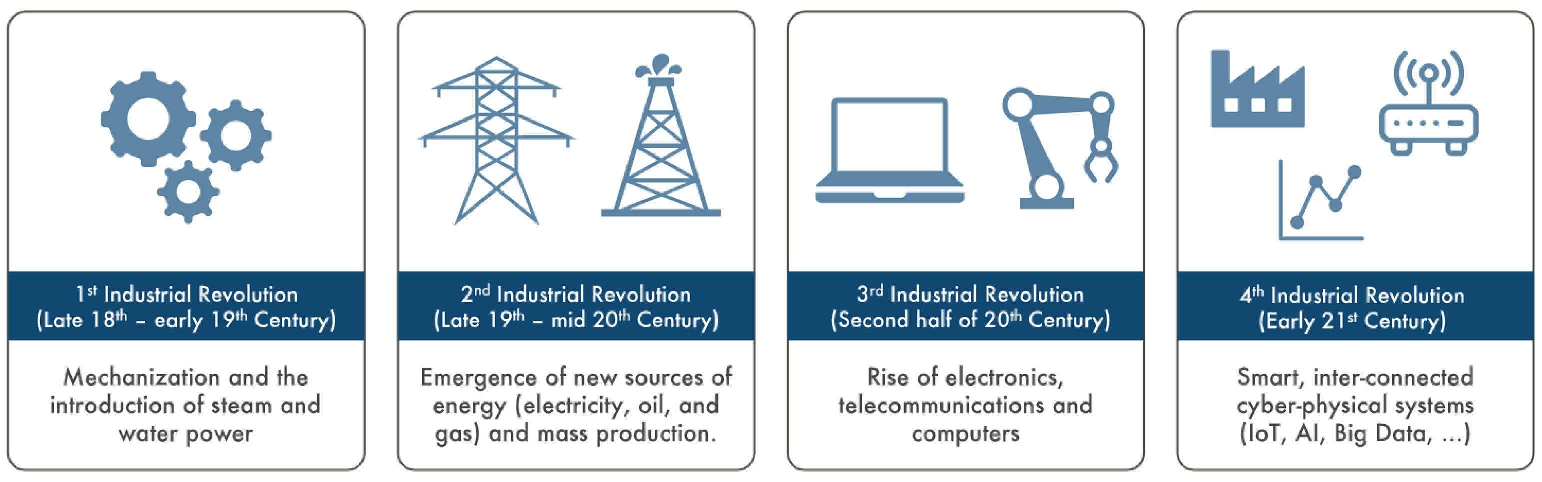
1. Automation and learning are the future



Four industrial revolutions

We are at the eve of a fourth industrial revolution in which **automation and intelligent systems** will be integrated deeply into our daily lives. This revolution will impact almost every aspect of our daily lives, ranging from self-driving cars, intelligent household appliances like smart phones, smart thermostats, and smart fridges, and intelligent manufacturing systems. All of these devices will be connected to the internet and will generate **enormous amounts of data**. This data offers tremendous opportunities when it is processed by **efficient algorithms** that enable devices to **improve and learn** from their past experience. The development of such algorithms is one of the most important mathematical challenges in the coming decades and the **Chair in Dynamics, Control, and Numerics** aims to contribute to this.

2. Automation in sheep herding

The trend for automation is also visible in the ancient profession of sheep herding. Farmers in Britain, Australia, and New Zealand are starting to replace their sheep dogs by **drones**. Some initial studies show great potential of this technology to reduce operation costs and the stress levels of the sheep. This is one of trends that inspired the **sheep herding game**.

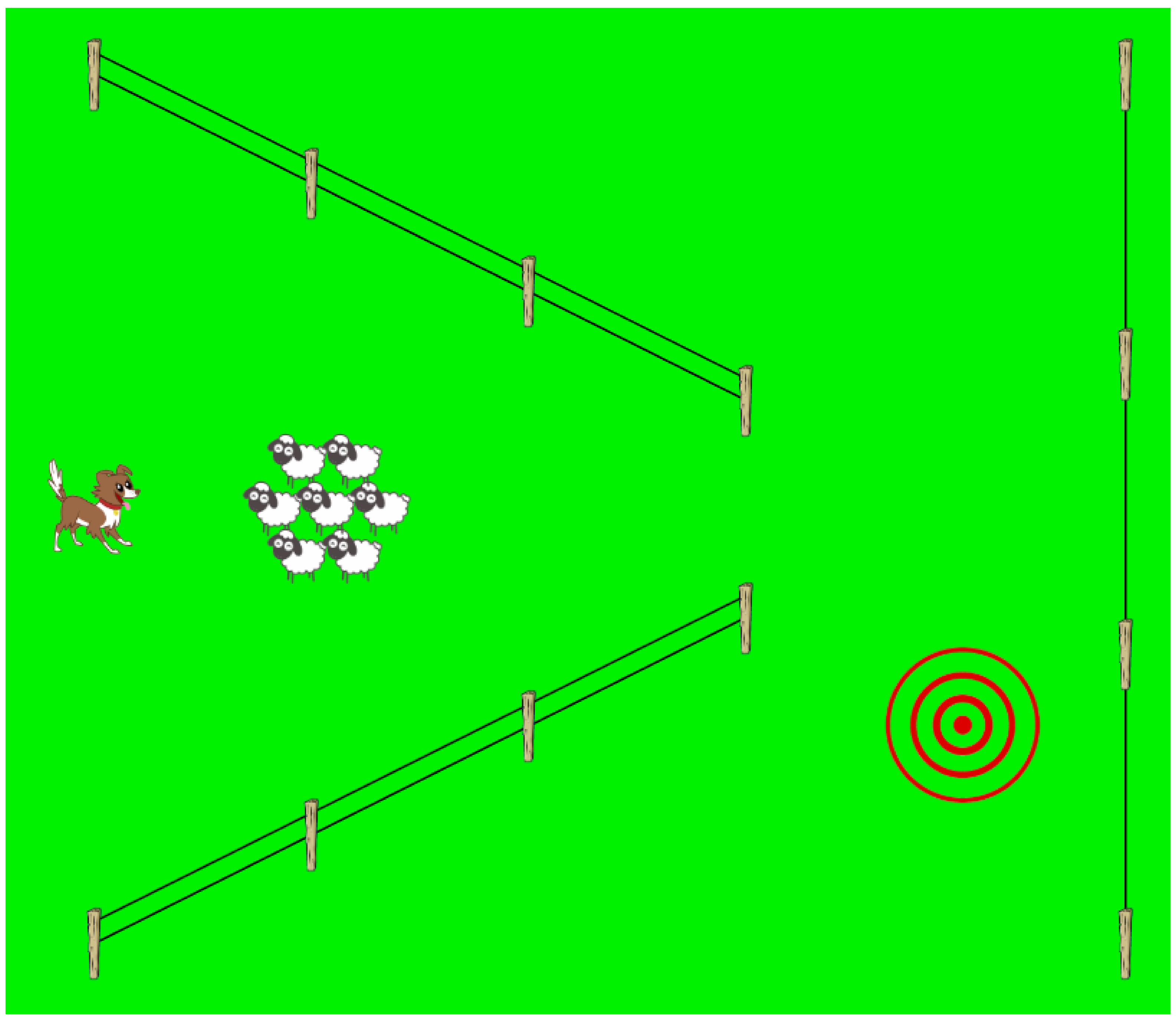
Sheep herding with drones in Wales [2]

3. The sheep herding game

Main rules:

- The dog should drive sheep to the target (red).
- You can steer the dog with the arrow keys.
- The sheep are afraid of the dog and of the fences.
- The farther the sheep are from the target, the faster the cost increases.
- The level is completed when all sheep stand still near the target.

Try it now for yourself and see if you can beat the algorithm!



A screen shot from the sheep herding game

4. The mathematics behind the game

In the game, you are in fact trying to solve a complex nonlinear optimal control problem: Find the control $\mathbf{u}(t)$ that minimizes

$$J = \frac{1}{N}\int_0^T \sum_{i=1}^N |\mathbf{x}_i(t) - \mathbf{x}_d|^2 \,\mathrm{d}t + \int_0^T |\mathbf{u}(t)|^2 \,\mathrm{d}t, \qquad (1)$$

subject to the dynamics

$$\dot{\mathbf{x}}_i(t) = \mathbf{v}_i(t), \qquad (2)$$

$$\dot{\mathbf{v}}_i(t) = \underbrace{\frac{1}{N-1}\sum_{j=1, j\neq i}^N \big(a(\mathbf{v}_j(t) - \mathbf{v}_i(t)) + g(\mathbf{x}_j(t) - \mathbf{x}_i(t))(\mathbf{x}_j(t) - \mathbf{x}_i(t))\big)}_{\text{Influence of the other sheep on sheep } i}$$
$$- \underbrace{f(\mathbf{y}(t) - \mathbf{x}_i(t))(\mathbf{y}(t) - \mathbf{x}_i(t))}_{\text{Influence of the dog on sheep } i} + \underbrace{\sum_{k=1}^{N_F} h_k(\mathbf{x}_i(t))}_{\text{Influence of the fences on sheep } i} \underbrace{-d\mathbf{v}_i(t)}_{\text{Sheep } i \text{ is lazy}}, \qquad (3)$$

$$\dot{\mathbf{y}}(t) = \mathbf{u}(t), \qquad (4)$$

and the initial conditions

$$\mathbf{x}_i(0) = \mathbf{x}_{i,0}, \qquad \mathbf{v}_i(0) = \mathbf{0}, \qquad \mathbf{y}(0) = \mathbf{y}_0. \qquad (5)$$

Here, $\mathbf{x}_i(t), \mathbf{v}_i(t) \in \mathbb{R}^2$ denote the position and velocity of sheep i, $\mathbf{y}(t) \in \mathbb{R}^2$ denotes the position of the dog, $\mathbf{x}_d$ is the position of the target, N is the number of sheep, N_F is the number of fences, $a = 1$, and $d = \frac{1}{3}$. The functions $g(\mathbf{x})$, $f(\mathbf{x})$, and $h_k(\mathbf{x})$ are given by

$$g(\mathbf{x}) = 2\left(1 - \frac{1}{3\sqrt{N}|\mathbf{x}|^2}\right), \qquad f(\mathbf{x}) = 4\exp(-8|\mathbf{x}|^2), \qquad (6)$$

$$h_k(\mathbf{x}) = 10\exp(-11|\mathbf{x} - \mathbb{P}_k\mathbf{x}|^2)(\mathbf{x} - \mathbb{P}_k\mathbf{x}), \qquad (7)$$

where $\mathbb{P}_k$ denotes the projection onto the k-th fence.

The optimal control is computed iteratively. That means that we start from an initial guess and gradually improve it based on the derivatives of J. This procedure can easily become very time consuming and the **development of efficient numerical algorithms** is an important scientific challenge.

The methods developed for this problem (see, e.g., [3,4]) are not only relevant for the **control of collective behavior**, but can also be applied in many other scientific contexts such as **the training of deep neural networks** and the **modeling and control of gas networks**.

Selected publications

[1] The DCN sheep herding game, https://github.com/danielveldman/sheep_herding_game (playing the game requires a MATLAB installation).

[2] North Wales Live (2019). **Drone shepherds who watch their flocks at height**.

[3] Ko, D., Zuazua, E. (2021). **Model predictive control with random batch methods for a guiding problem.** Math. Models Methods Appl. Sci., Vol. 31, No. 8.

[4] Veldman, D.W.M., Zuazua, E. (2022) **A framework for stochastic time-splitting in linear quadratic optimal control.** Numerische Mathematik.

https://dcn.nat.fau.eu/
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