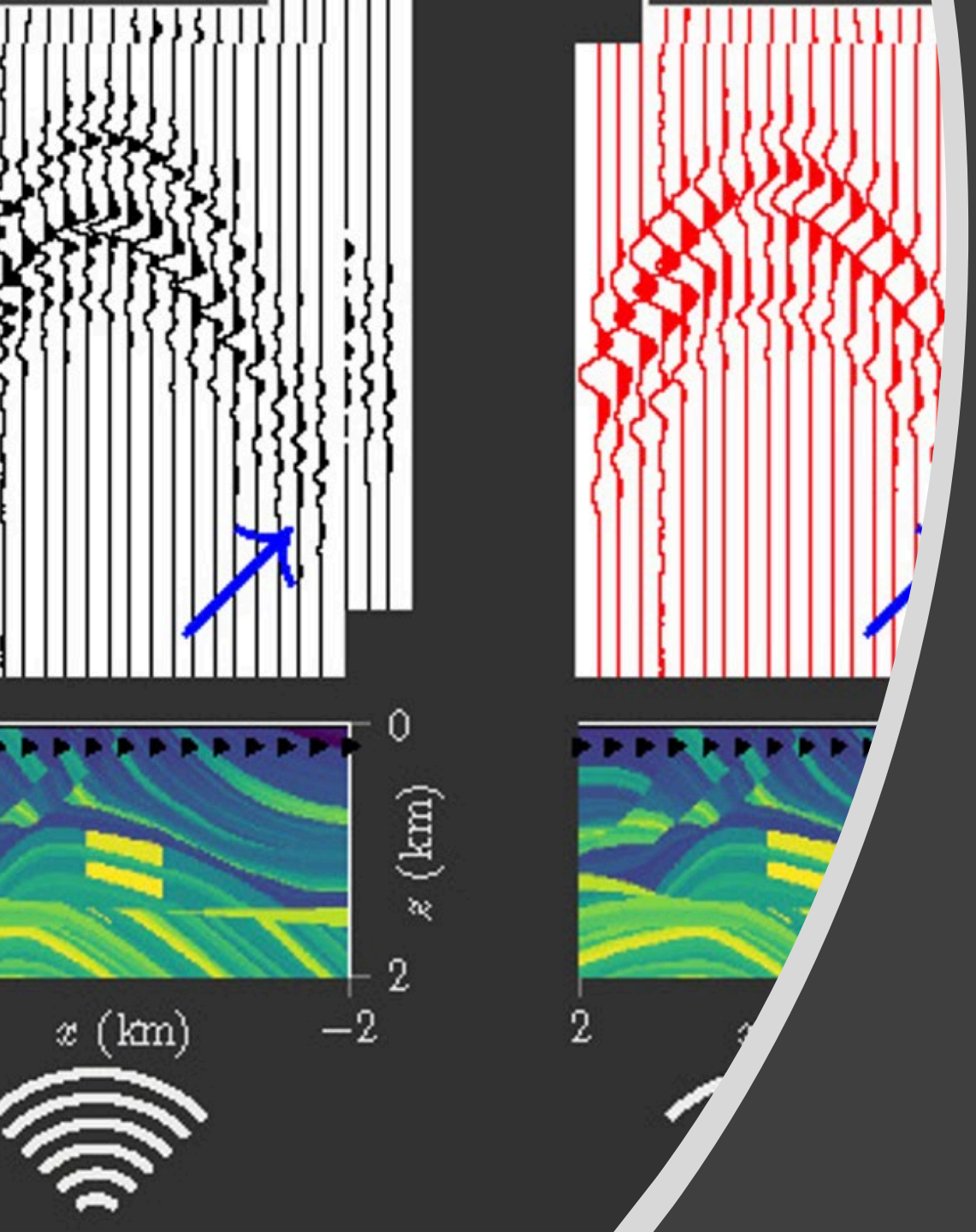


Deep redatuming for PDE and inverse problems

Laurent Demanet

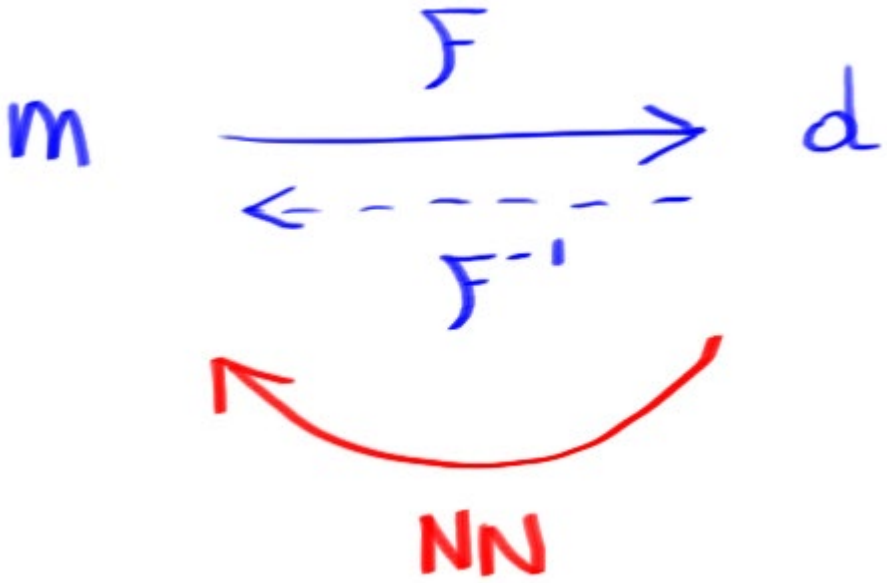
Math and EAPS, MIT

With Pawan Bharadwaj (IIS), Matt Li (MIT), Hongyu Sun (Caltech), Brindha Kanniah (MIT)

FAU-AvH seminar, Erlangen

June 10, 2022

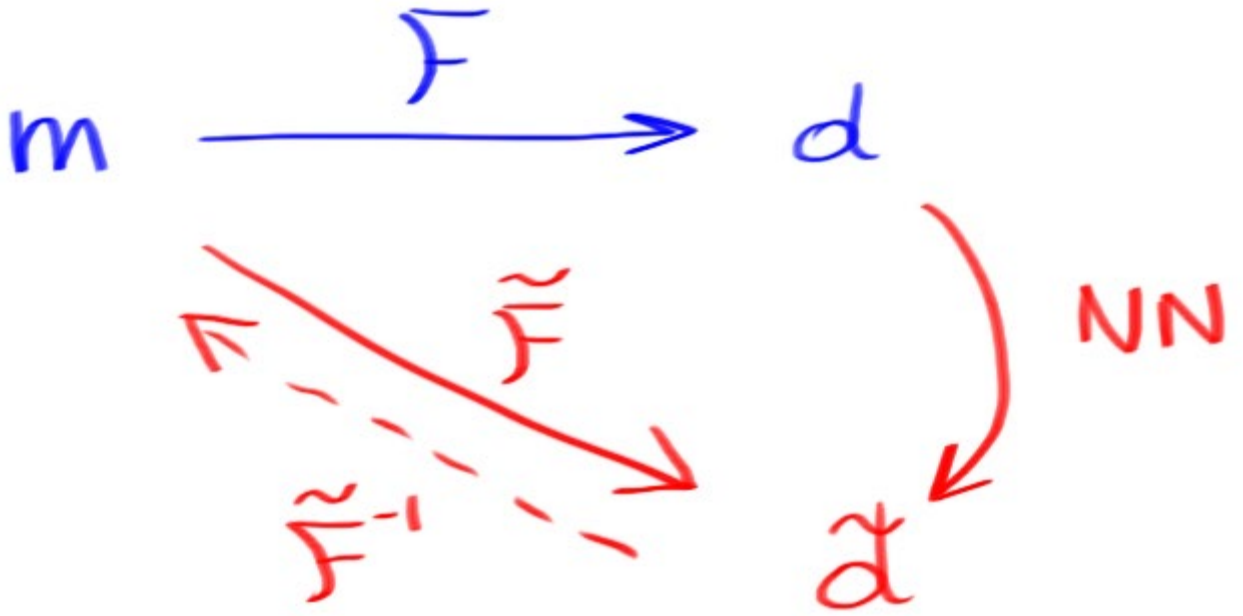
Neural networks for inverse problems



OK for small scale problems

Invert $\mathcal{F}$ (train on simulations) -- or bypass $\mathcal{F}$ (train on real data)

Neural networks for inverse problems

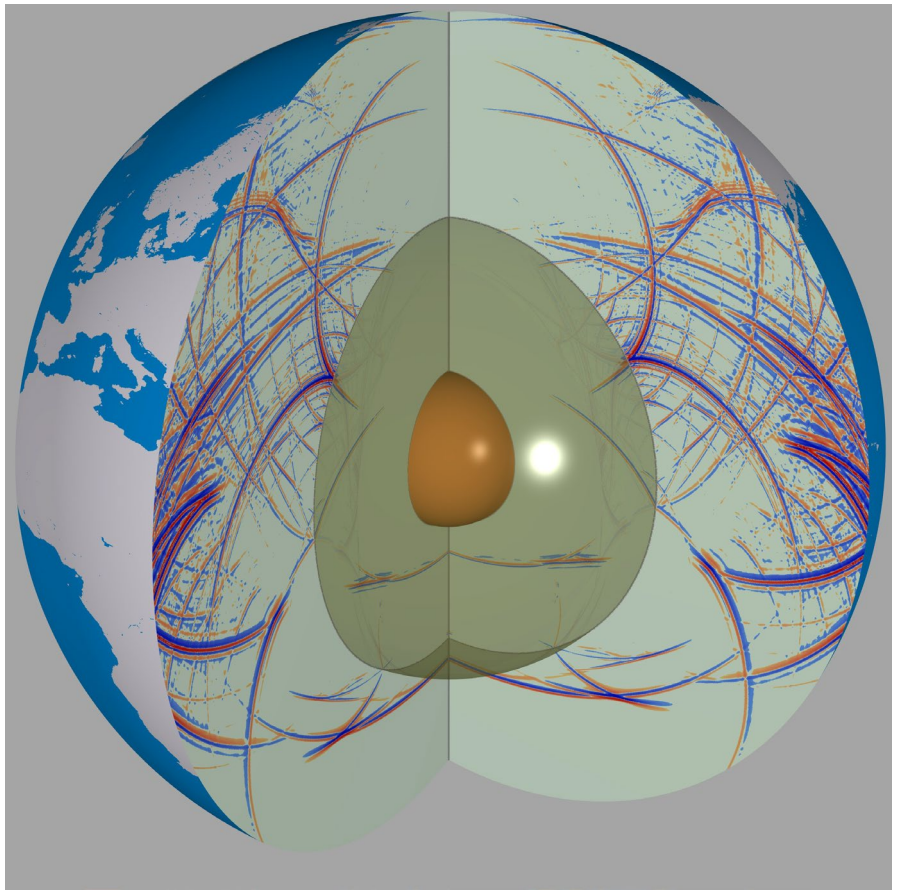


Auxiliary data extension task

More robust or favorable inversion

Bridge real vs synthetic divide

Context: seismic inversion



Credit: TU Muenchen

Initialization (velocity model building) by deep nets in Araya et al (2018)

Overview

1. Bandwidth extension (w/ Hongyu Sun)

2. “Physics Swap” SymAE (w/ Pawan Bharadwaj and Matt Li)

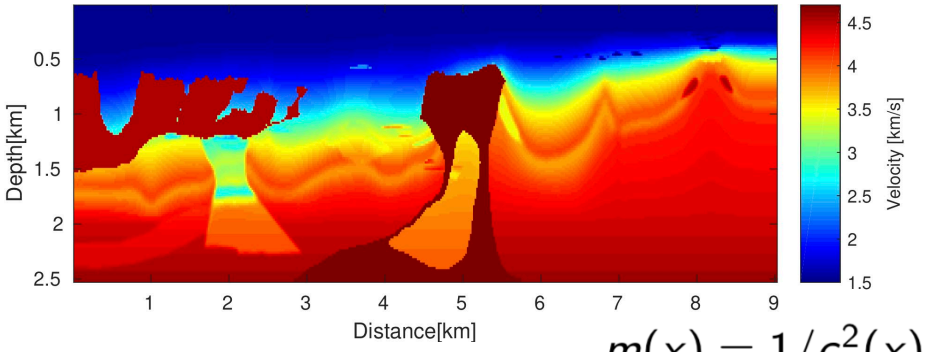
Direct vs inverse problem

Forward problem

Given f, ω, and m, find $u_{\omega,f}$ s.t.

$$(\Delta + \omega^2 m(x))u_{\omega,f}(x) = f(x)$$

Then $d_{r,\omega,f} = u_{\omega,f}(x_r)$.



$m(x) = 1/c^2(x)$

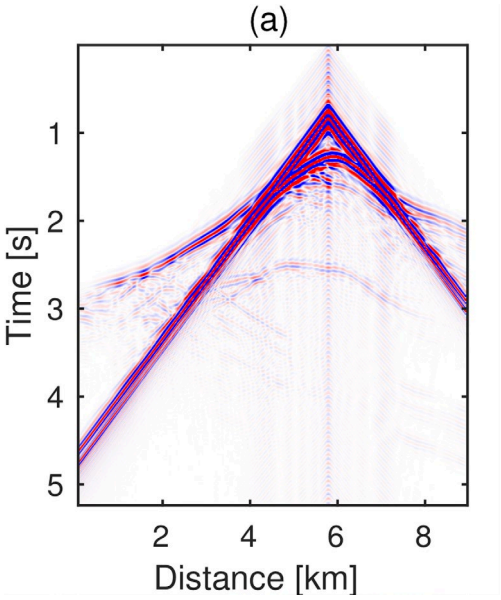
Inverse problem

Given several f, several ω, and data samples viewed as

$$d_{r,\omega,f} = u_{\omega,f}(x_r),$$

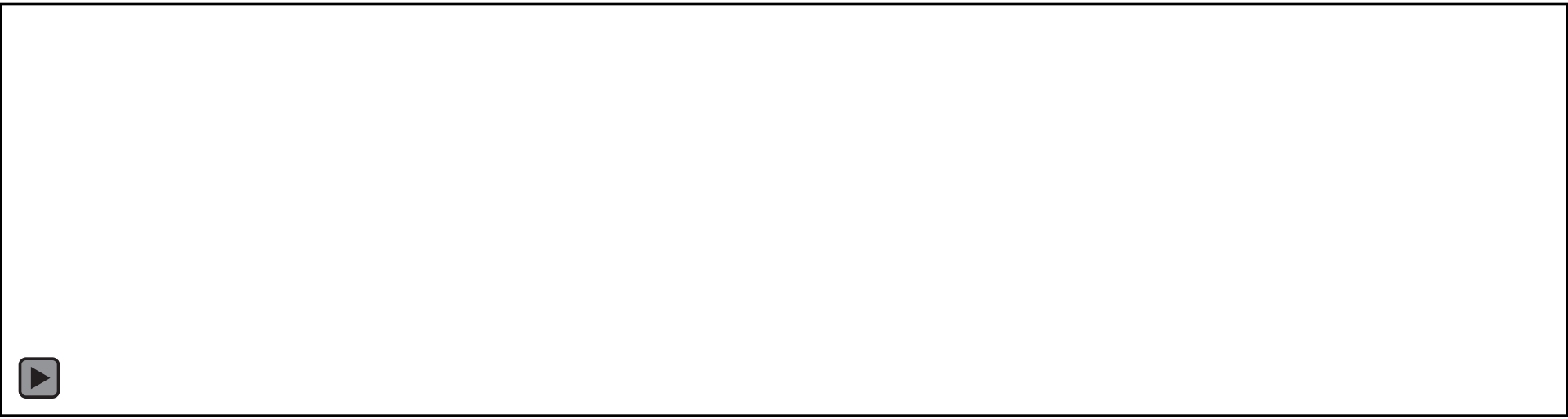
find the rest of $u_{\omega,f}$, and m, s.t.

$$(\Delta + \omega^2 m(x))u_{\omega,f}(x) = f(x)$$

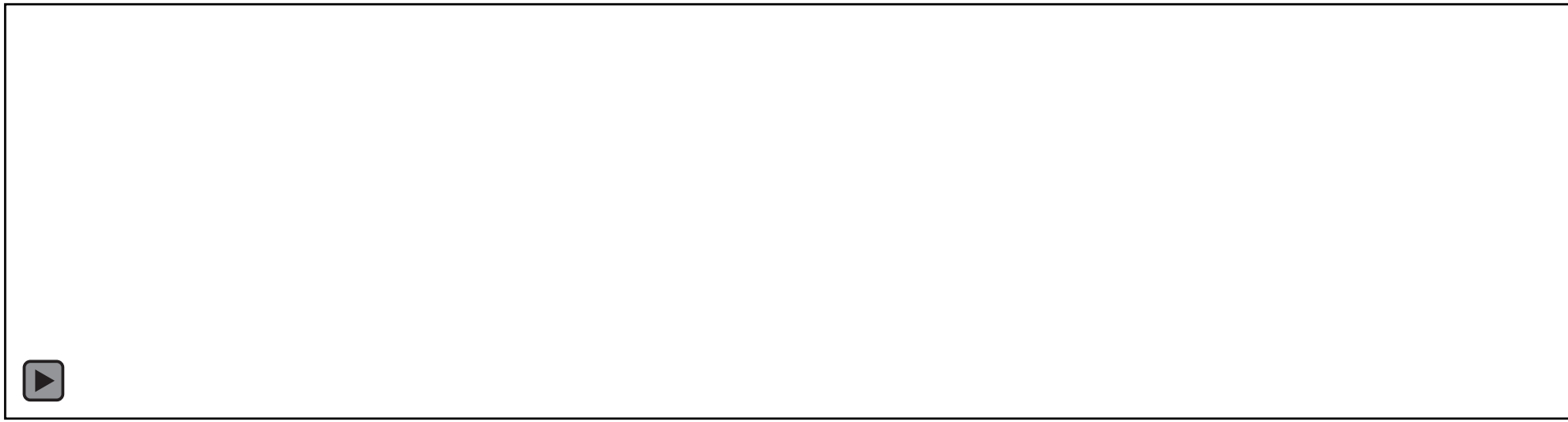


$d_{r,\omega,f}$ in the time domain

Running the forward model

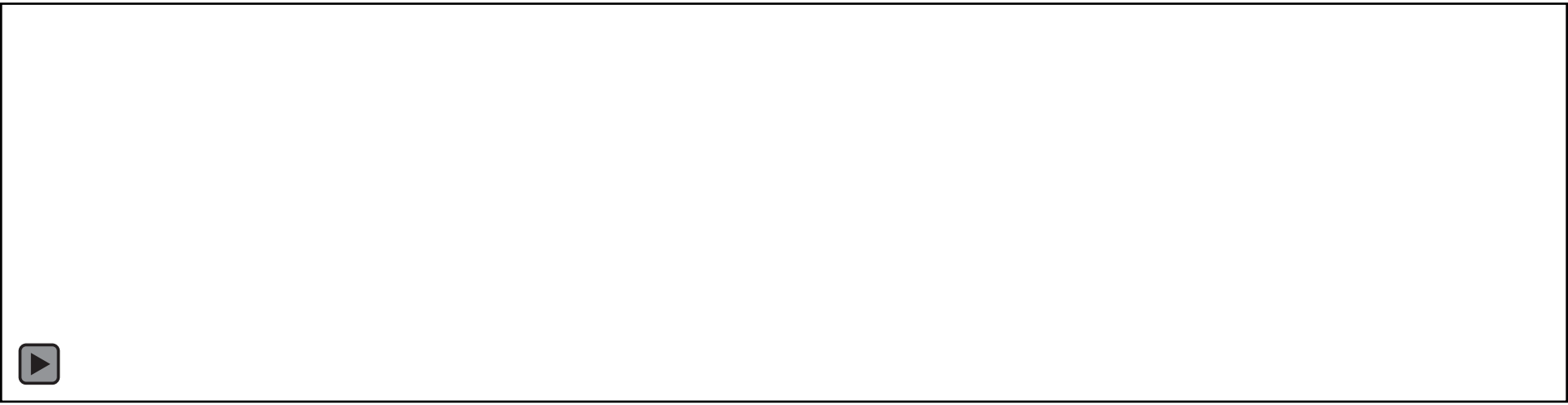


Inverting for m, good initial guess



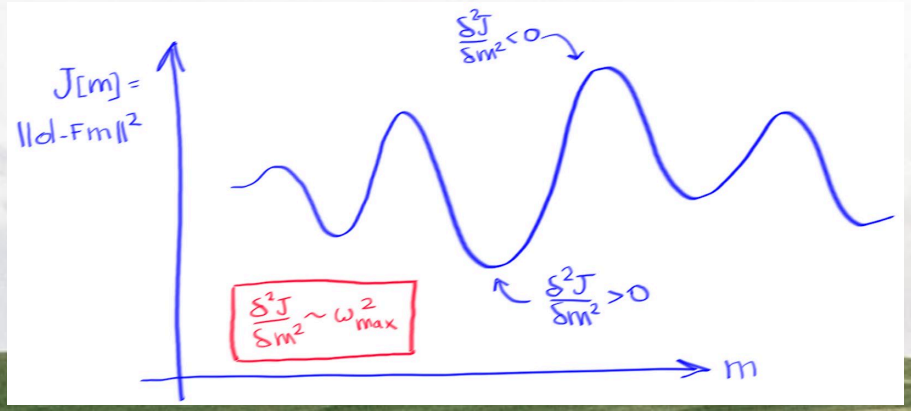
Data in [0.6, 20] Hz

Inverting for m, bad initial guess

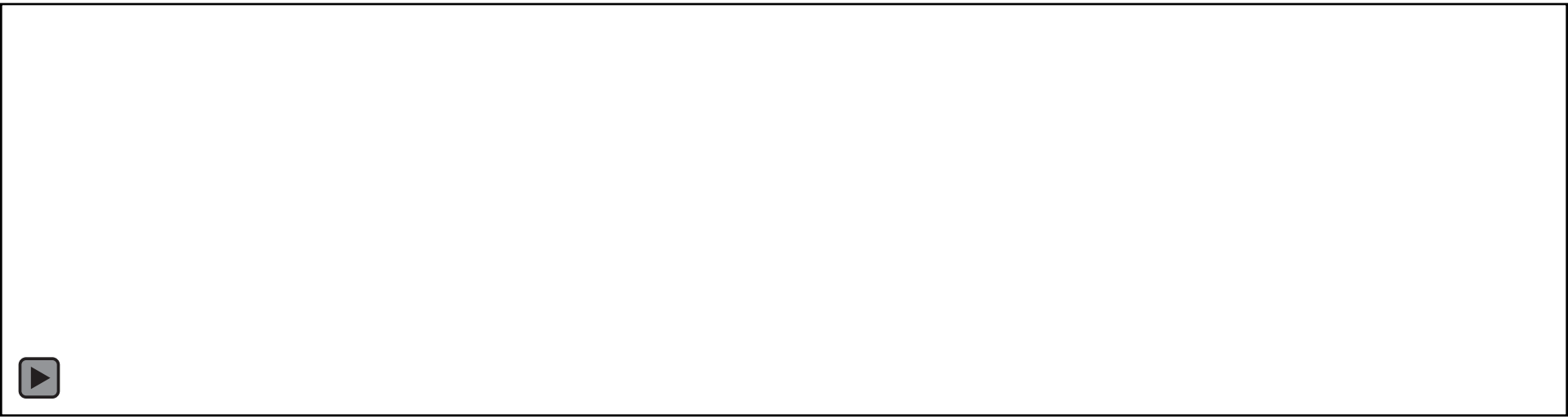


Data in [0.6, 20] Hz

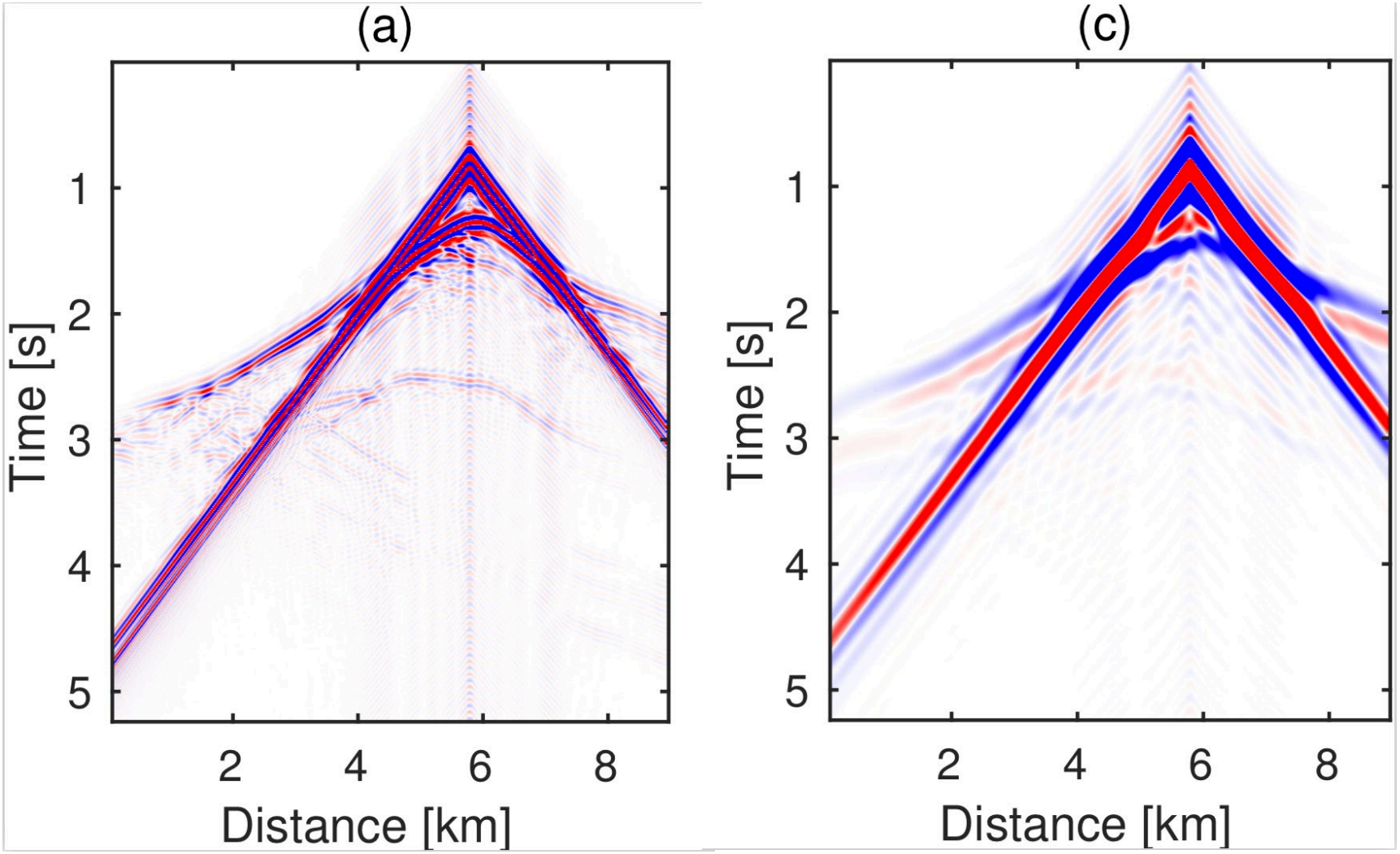
Solving for m is hard!



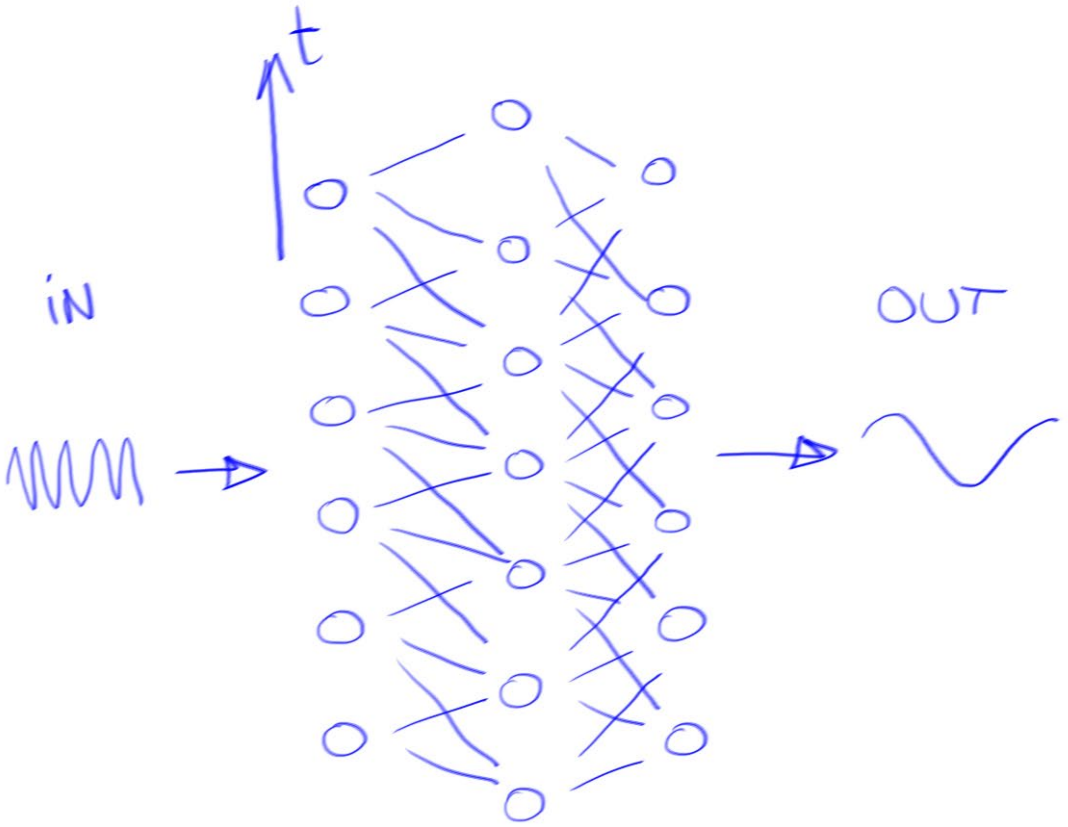
Running the forward model (low freq)



Left – resulting data at the surface, high frequencies
Right – resulting data at the surface, low frequencies

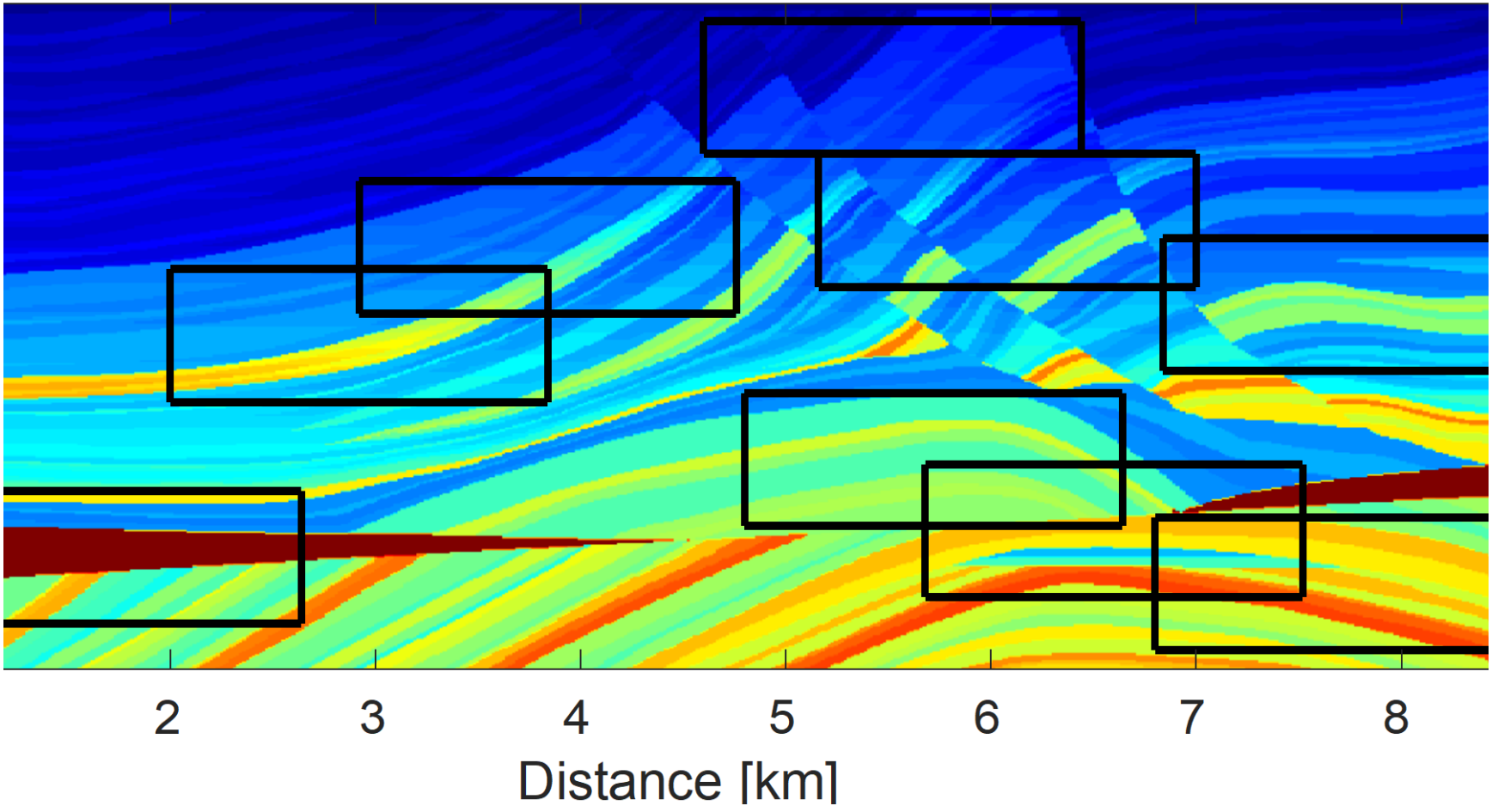


A deep net for frequency extrapolation

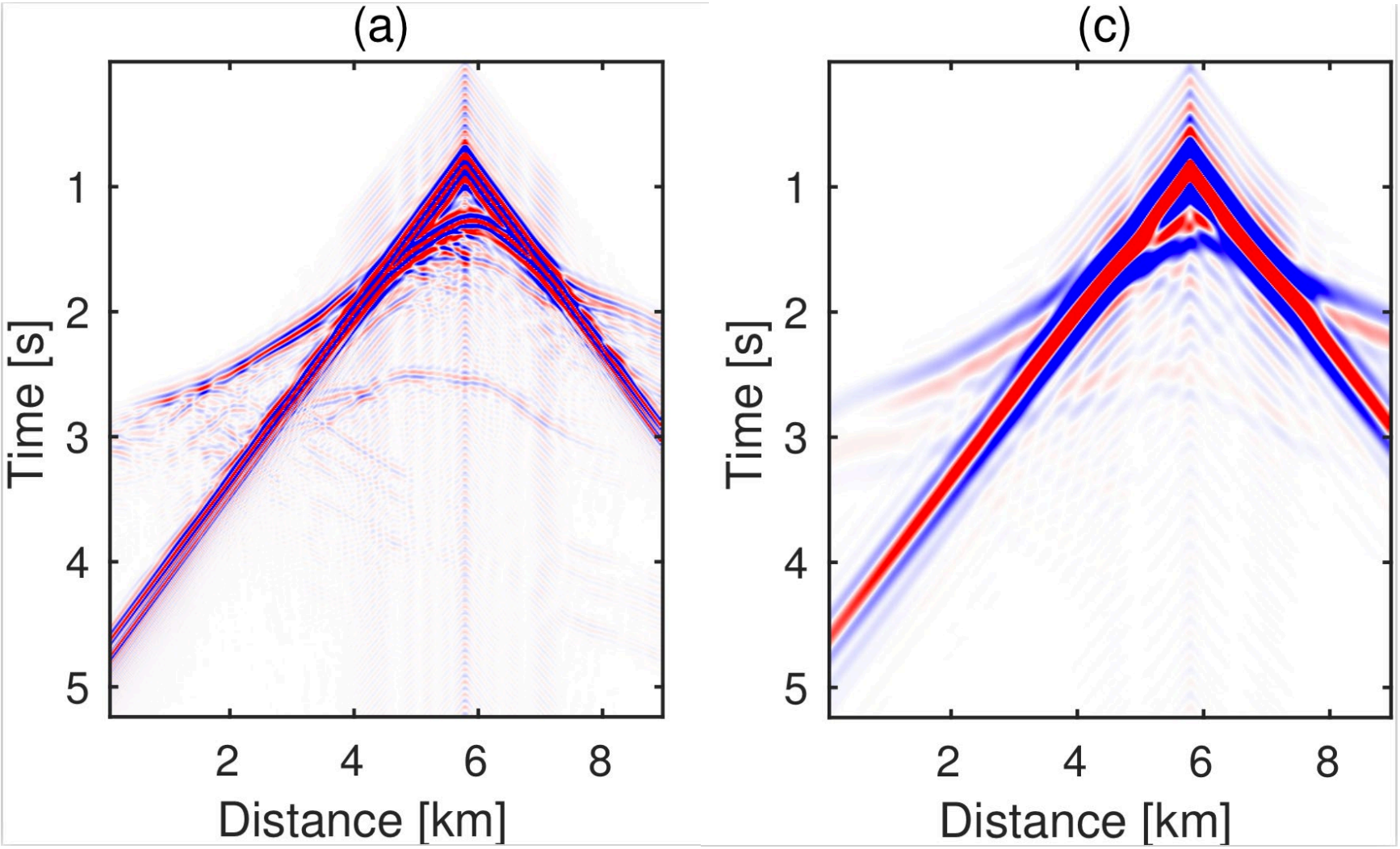


Convolutional, wide filters, width ~ 100, depth 5

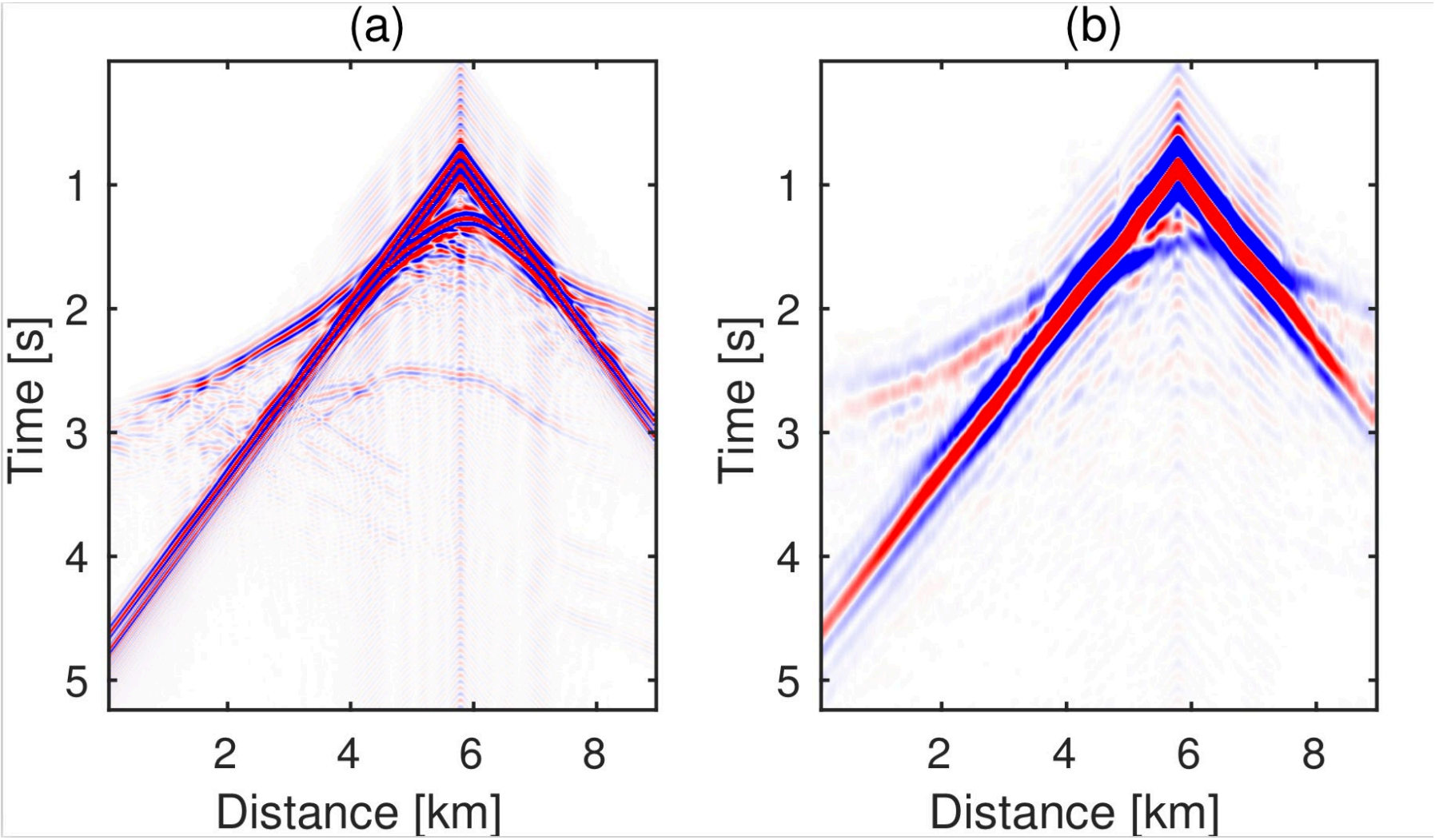
Training from 9 “fake Earths”



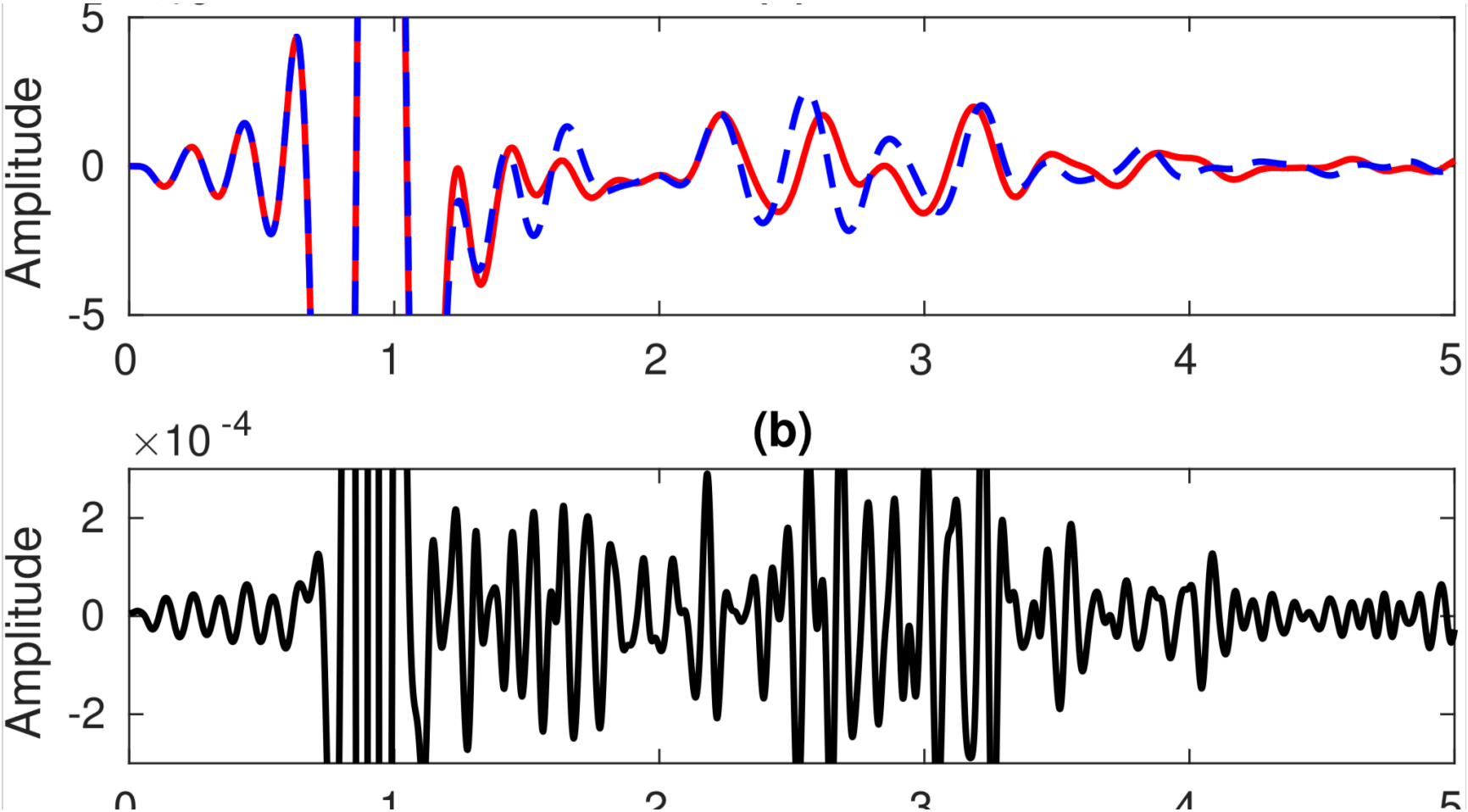
True vs extrapolated LF



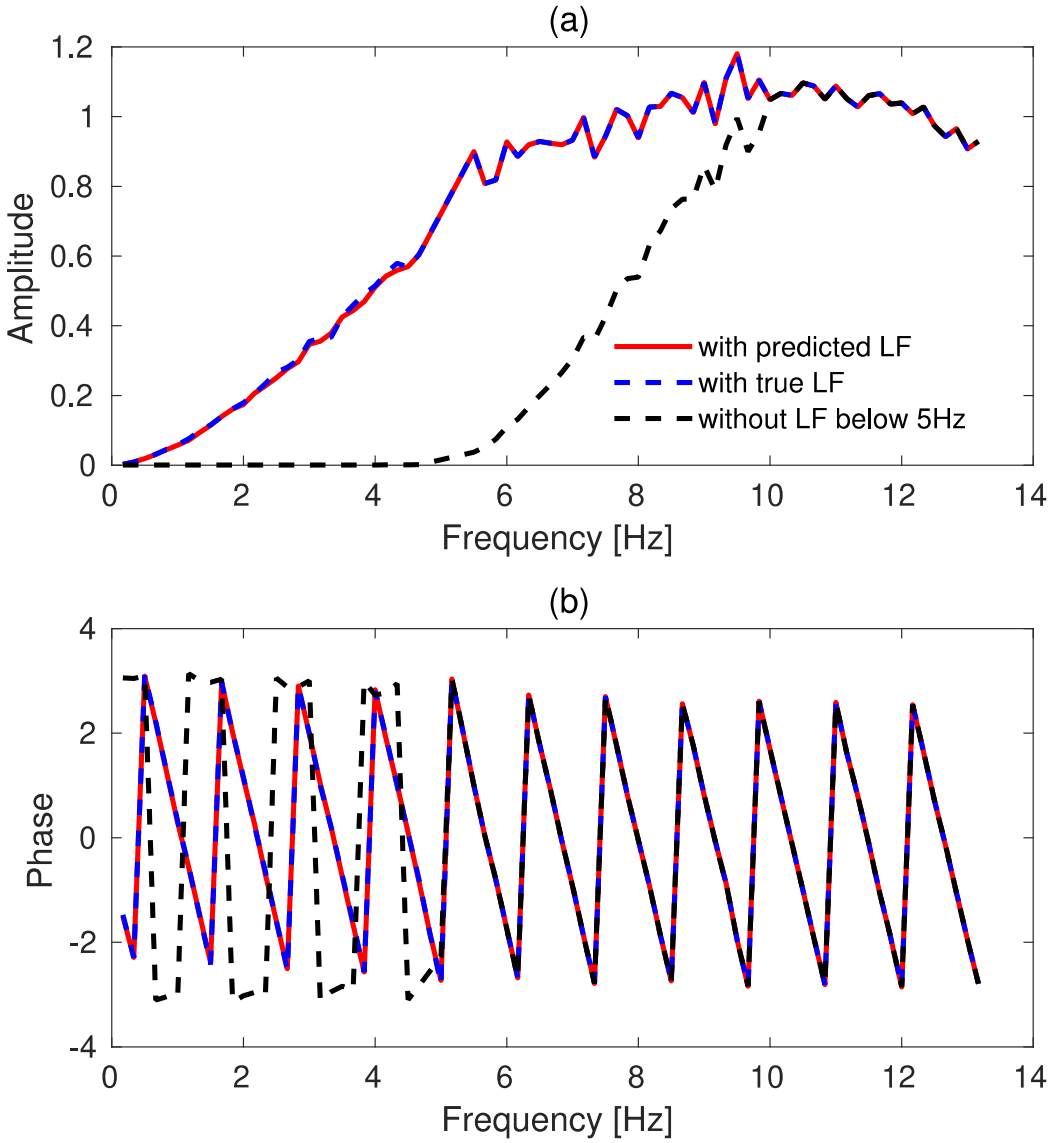
True vs extrapolated LF



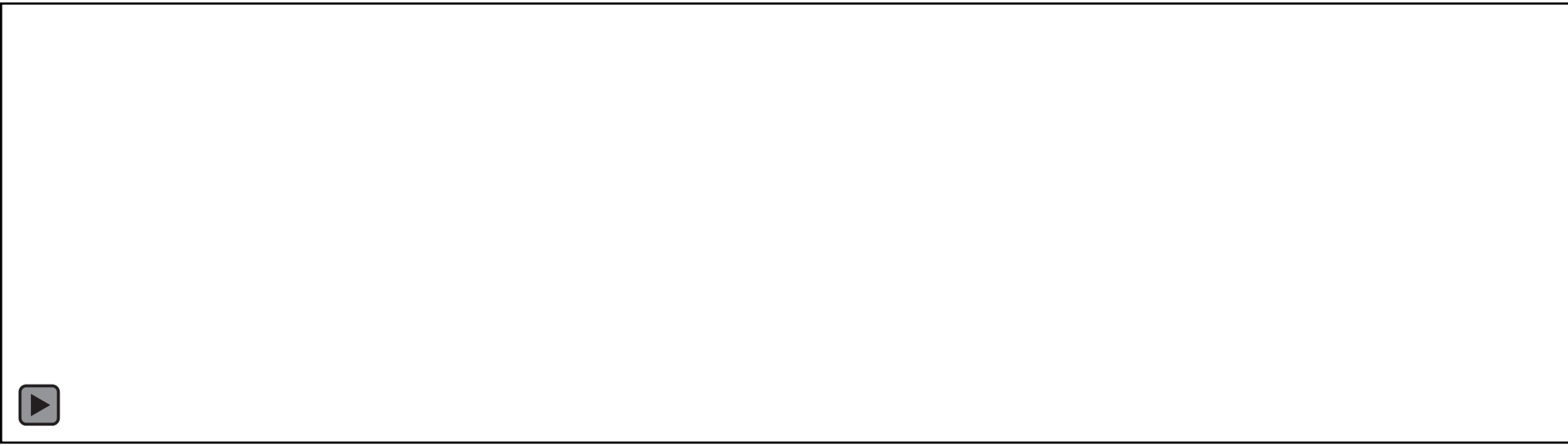
In the time domain



In the frequency domain

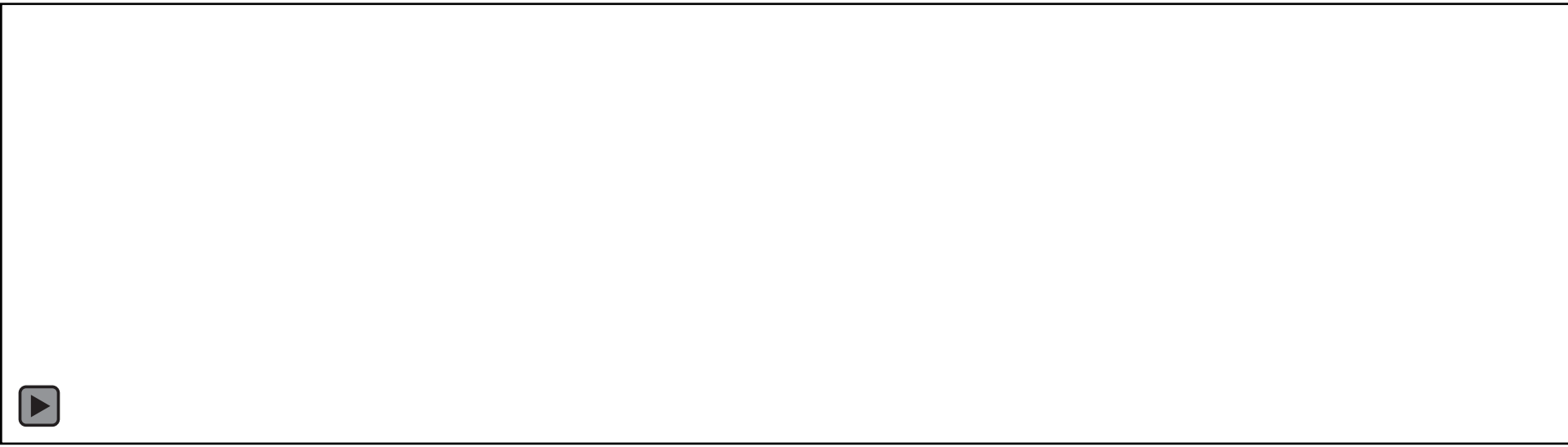


Inverting for m, bad initial guess



Data in [0.6, 20] Hz

Inverting for m, bad initial guess



Data in [0.6, 20] Hz with
extrapolation down to 0.3 Hz

Overview

1. Bandwidth extension (w/ Hongyu Sun)

2. “Physics Swap” SymAE (w/ Pawan Bharadwaj and Matt Li)

Data

Black box predictor

What can a deep net know?

Simulations

Black box model replicator

Data

Black box predictor

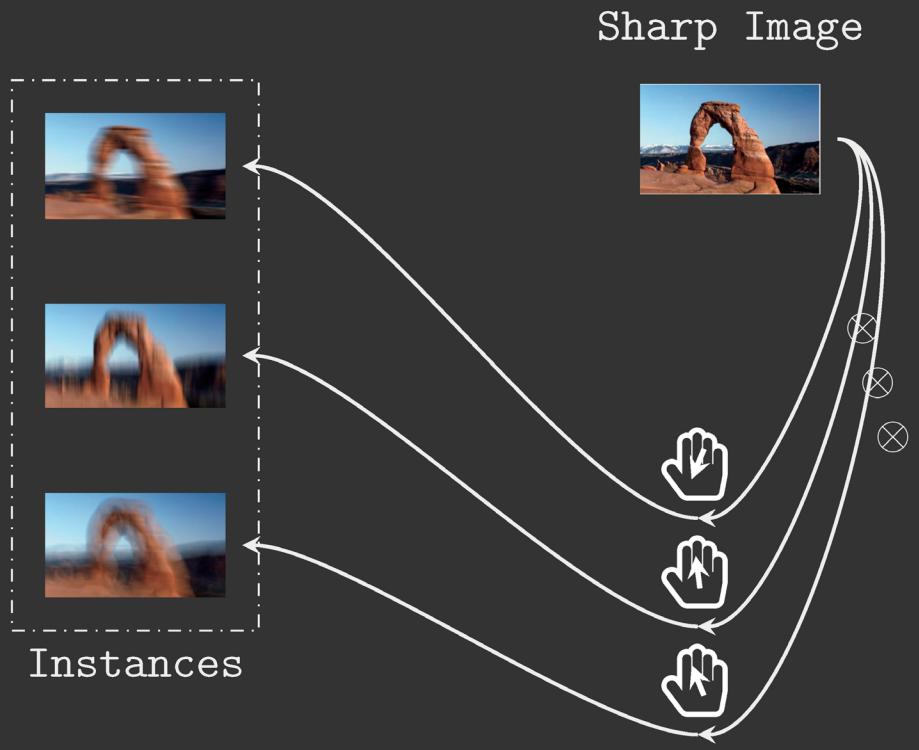
What can a deep net know?

Disentangle explanatory components

Simulations

Black box model replicator

>>> WHAT: Estimate *Similarity* or *Coherency* Among Instances



→ Coherent Component

which is coherent across all the instances

→ Known Linear Operation ⊗

→ Dissimilar Component

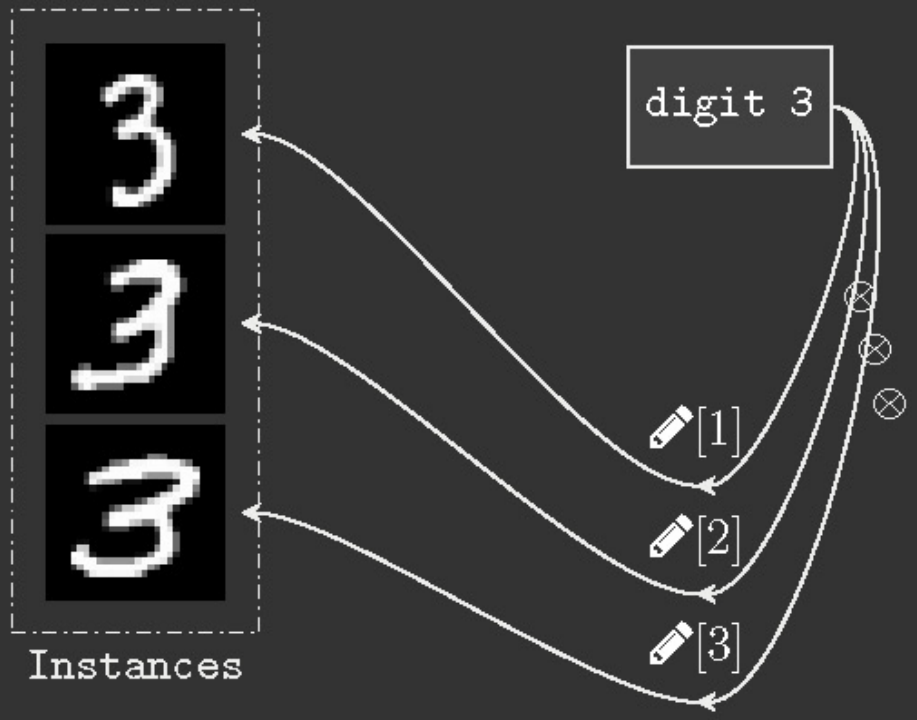
which is dissimilar among instances

IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 67, NO. 12, JUNE 15, 2019

Focused Blind Deconvolution

Pawan Bharadwaj, Laurent Demanet, and Aimé Fournier

>>> WHAT: Estimate *Similarity* or *Coherency* Among Instances



→ Coherent (G)

3; digit information is similar in all images

→ Unknown Operation $\otimes$

→ Dissimilar (Nuisance; W)

✎; writing style and orientation are dissimilar among instances

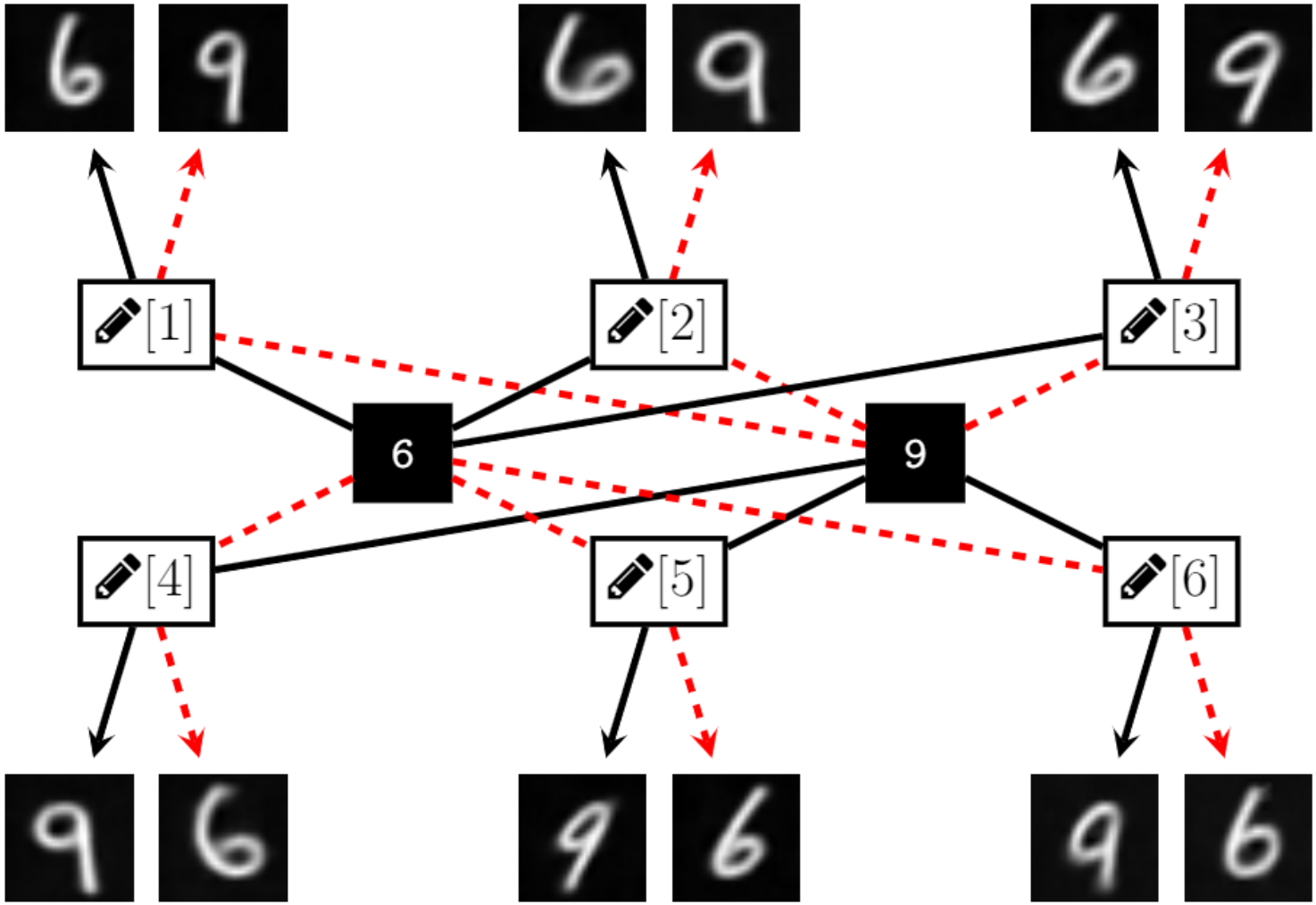
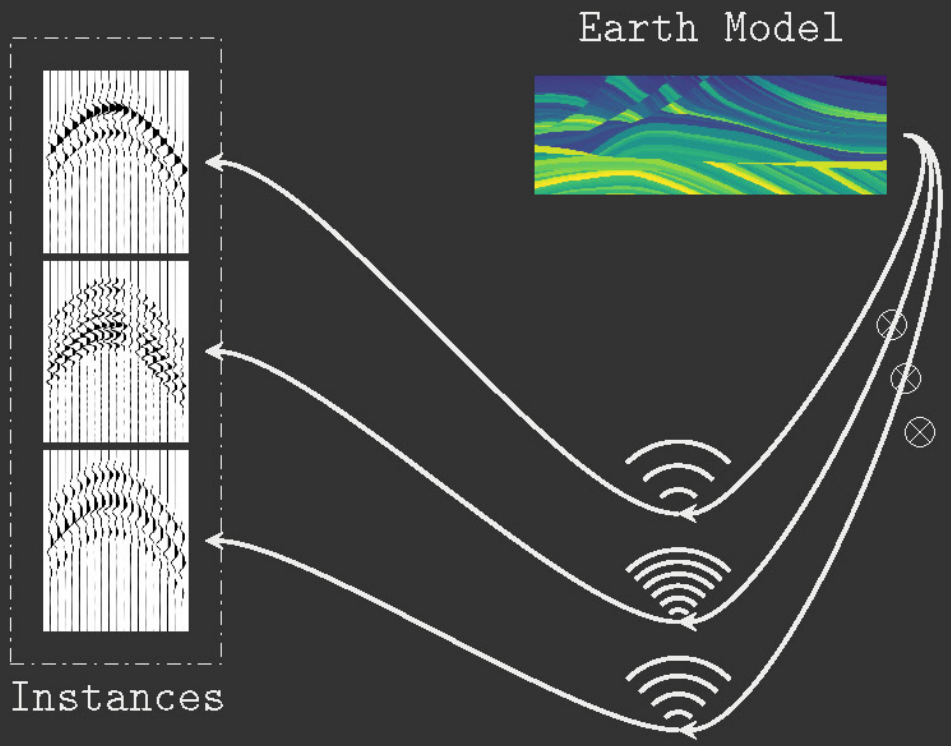


Fig. 1: In the MNIST experiment, the SIMO system responds to the input digit information (here, 6 and 9) by producing multiple dissimilar hand-written images as channel outputs. Here, six channel outputs are plotted. The first three channels didn't respond to 9, however SymAE produces their virtual outputs (dashed lines) that have identical writing style as in the true outputs. Similarly, virtual channel outputs of the last three channels are also plotted.

>>> WHAT: Estimate *Similarity* or *Coherency* Among Instances



→ Coherent (G)
medium effects are similar in all passive shot gathers

→ Unknown Operation $\otimes$

→ Dissimilar (Nuisance; W)
source mechanism, signature and position are dissimilar among instances

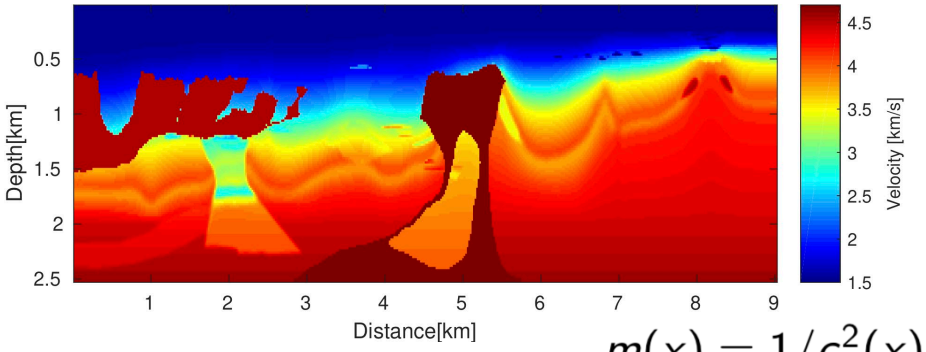
Direct vs inverse problem

Forward problem

Given f, ω, and m, find $u_{\omega,f}$ s.t.

$$(\Delta + \omega^2 m(x))u_{\omega,f}(x) = f(x)$$

Then $d_{r,\omega,f} = u_{\omega,f}(x_r)$.



$m(x) = 1/c^2(x)$

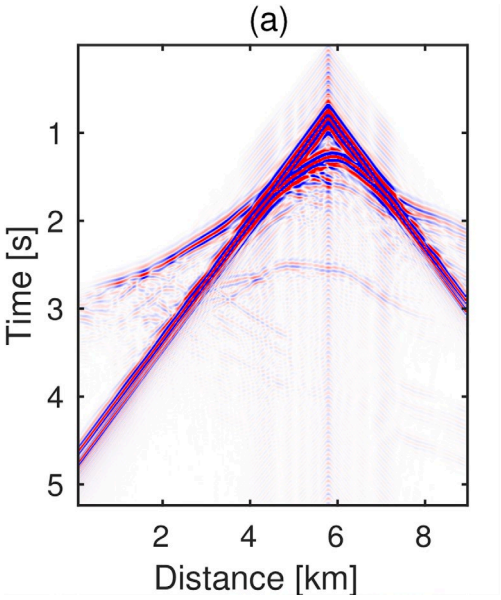
Inverse problem

Given several f, several ω, and data samples viewed as

$$d_{r,\omega,f} = u_{\omega,f}(x_r),$$

find the rest of $u_{\omega,f}$, and m, s.t.

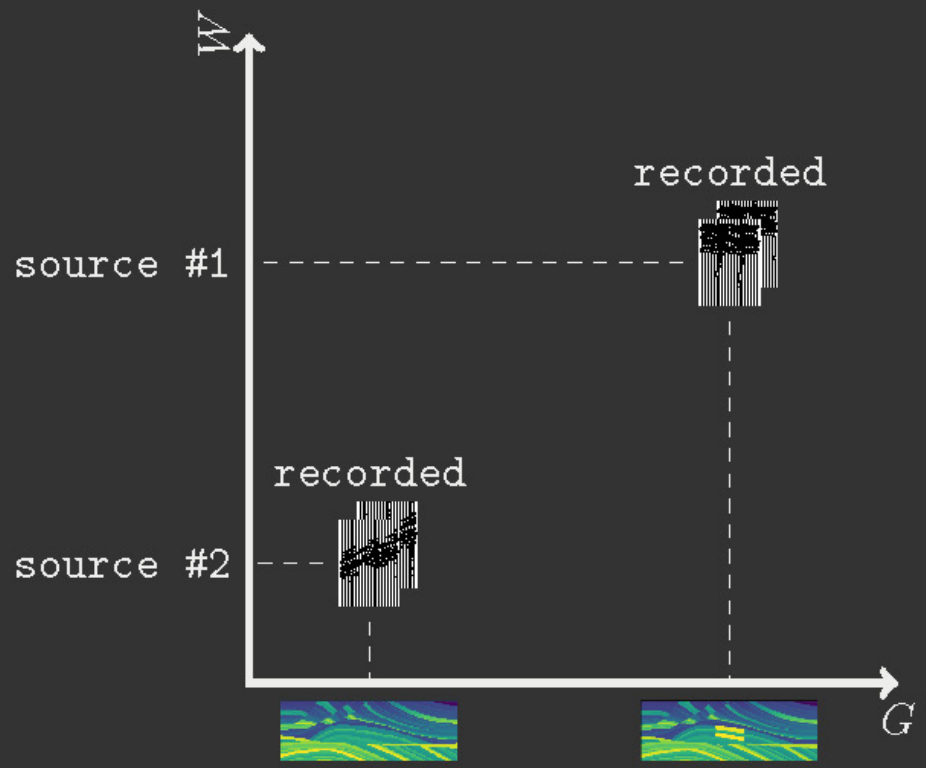
$$(\Delta + \omega^2 m(x))u_{\omega,f}(x) = f(x)$$



$d_{r,\omega,f}$ in the time domain

>>> SymAE's Latent-space Structure

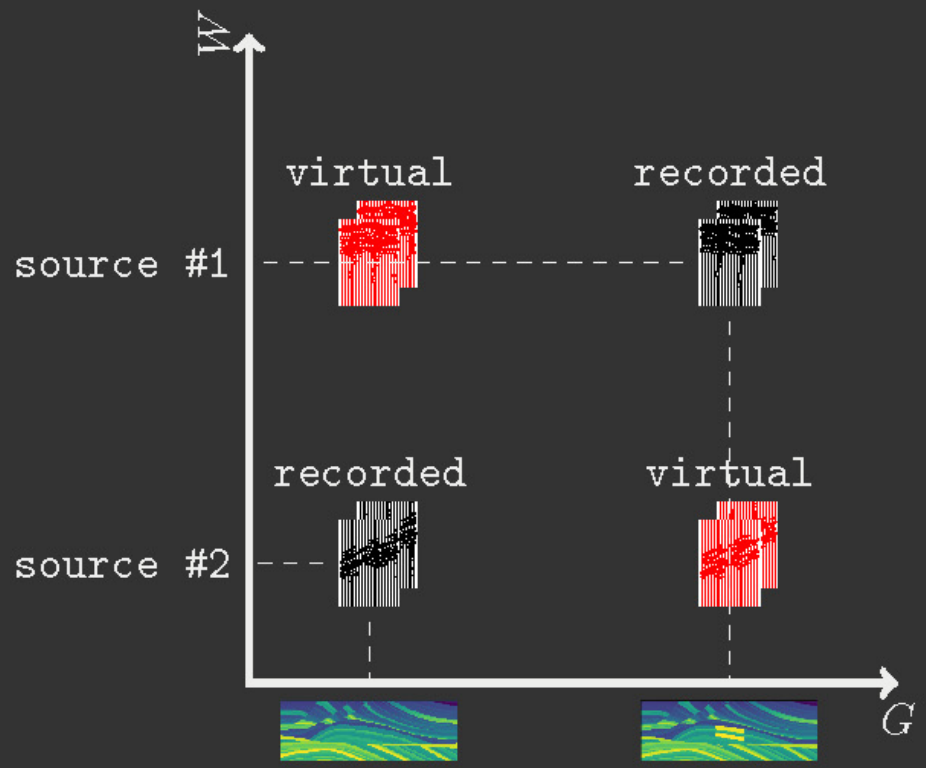
coherent G	$\rightarrow$ medium effects
nuisance W	$\rightarrow$ passive sources



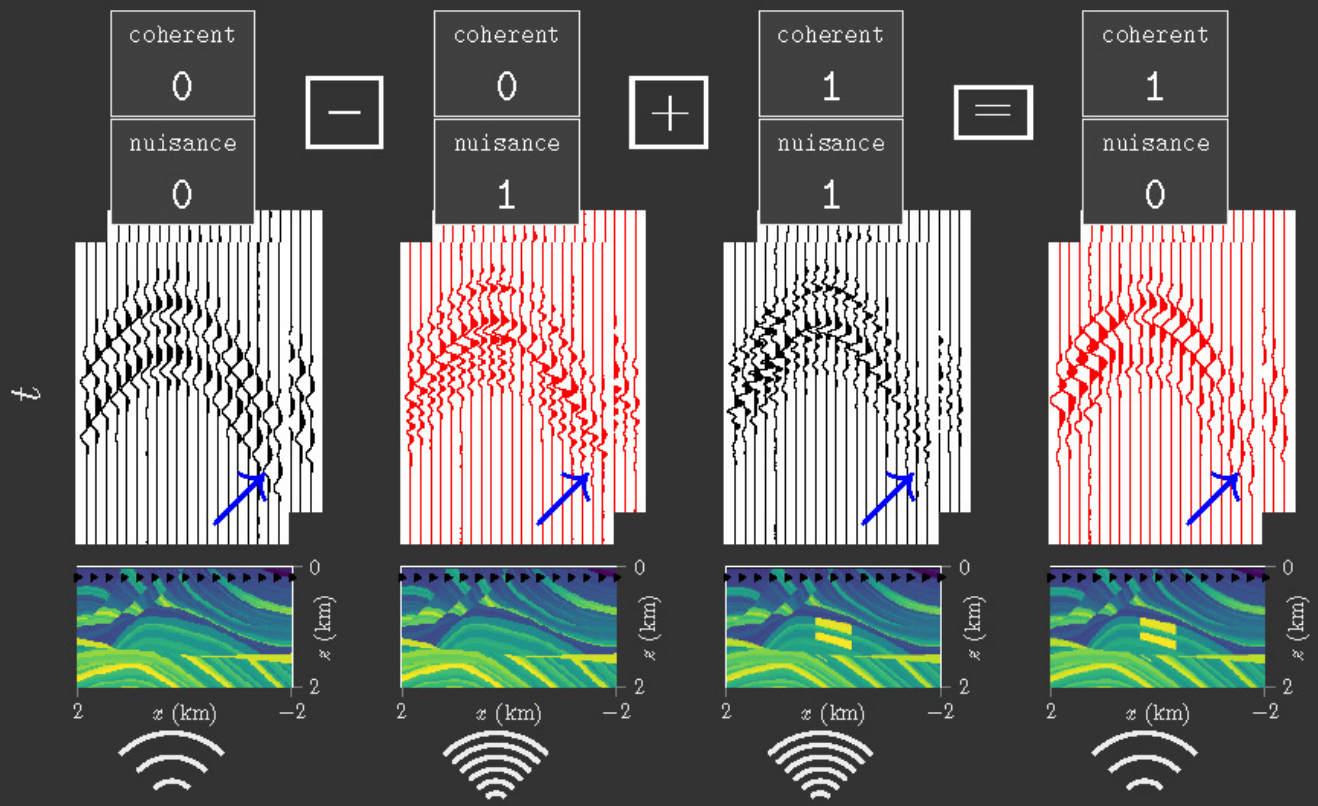
>>> SymAE's Latent-space Structure

coherent G → medium effects

nuisance W → passive sources



>>> WHY: Redatum To Produce Virtual Gathers!



Solution: SymAE produces virtual baseline sources during monitoring

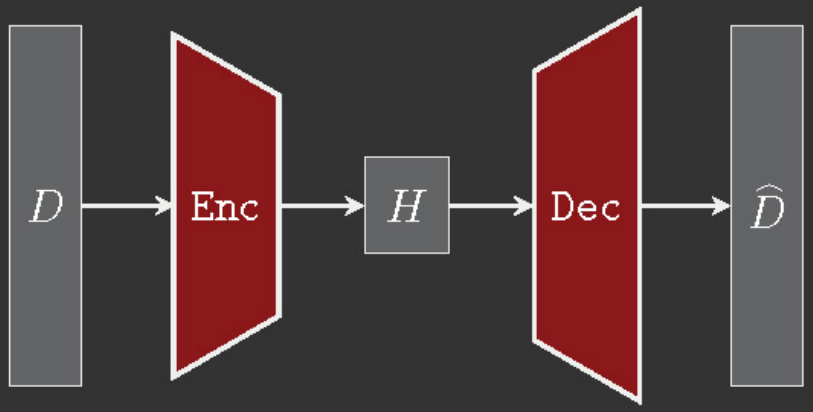
>>> Autoencoder[1]

Encoder

$$H_i = \texttt{Enc}(D_i)$$

Decoder

$$\widehat{D_i} = \texttt{Dec}(H_i)$$



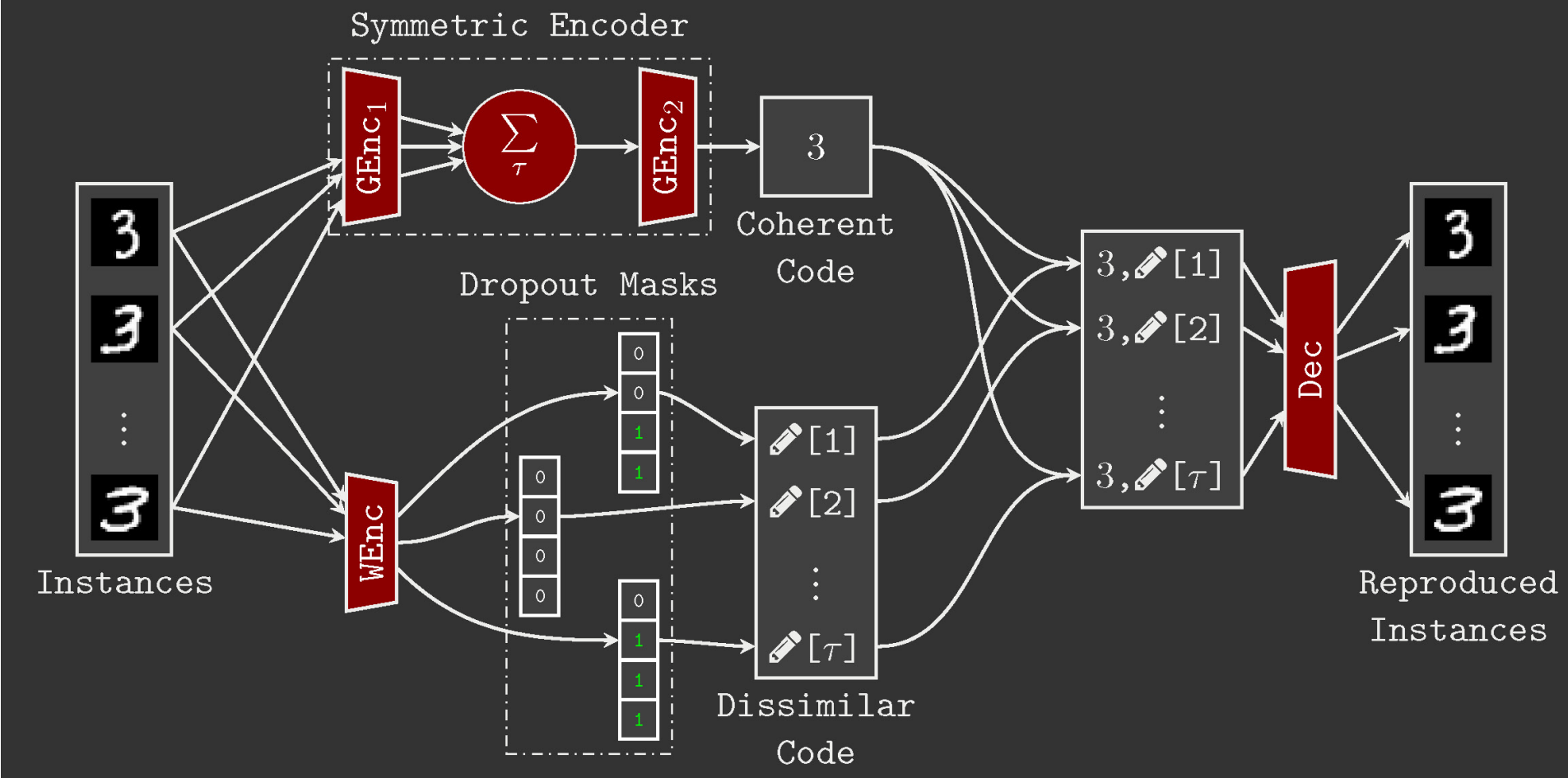
Training

$$\texttt{Enc}, \texttt{Dec} = \underset{\texttt{Enc}, \texttt{Dec}}{\arg\min} \sum_i \| D_i - \texttt{Dec}(\texttt{Enc}(D_i)) \|^2$$

---

[1] Kramer, M. A., 1991, Nonlinear principal component analysis using autoassociative neural networks, AIChE Journal.

>>> HOW: SymAE's Network Architecture



Why deep nets… and why not

New inversion/inference tools when models are insufficient

What are we giving up?

- interpretability
- guarantees (generalizablility)
- Science!
- … still not automated

Why does this work?

There is an underlying model of the form

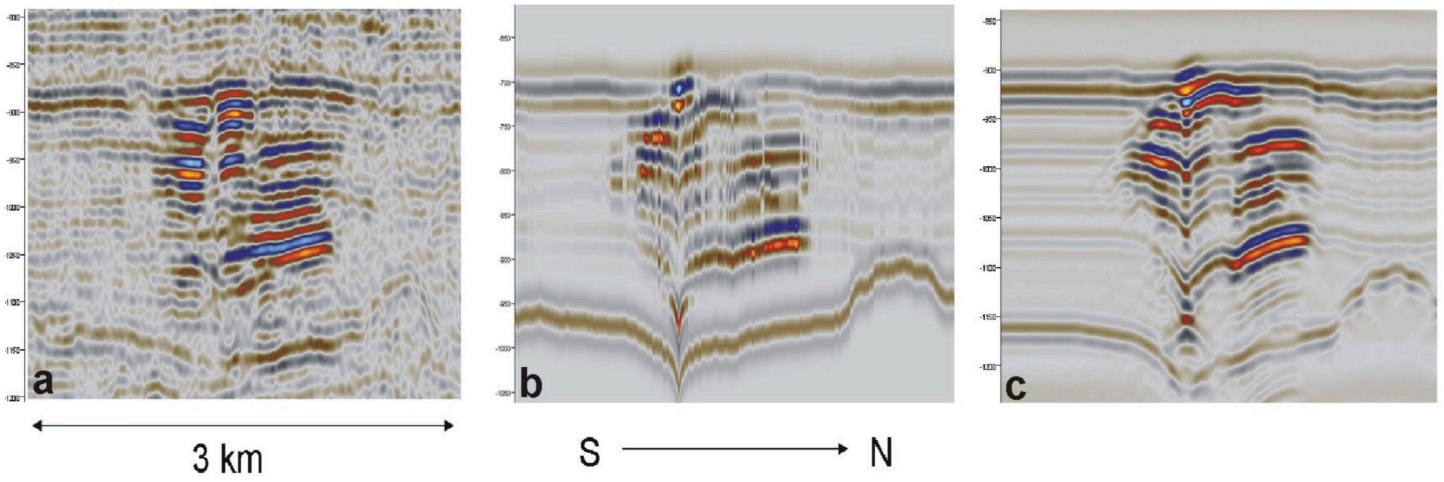
$$\mathbf{d}_{ij} = \mathcal{F}(x_i, y_j)$$

Symmetric under permutations

"Latent rank-1"

The network's explanation is

$$\mathbf{d}_{ij} = \mathcal{G}(h_i^{(x)}, h_j^{(y)})$$

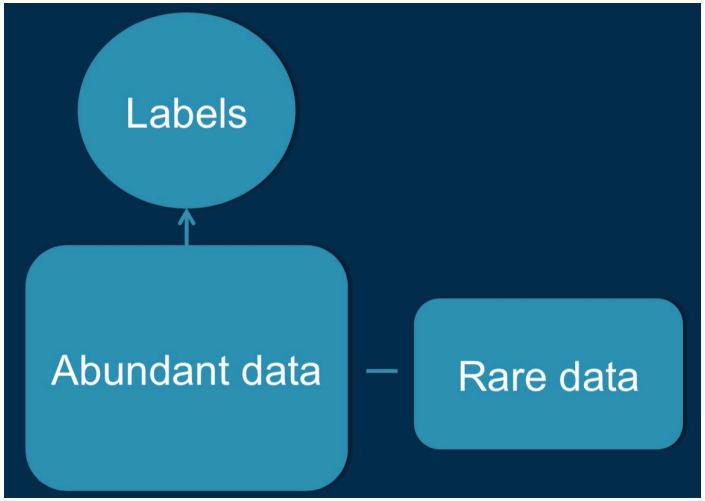


Outlook

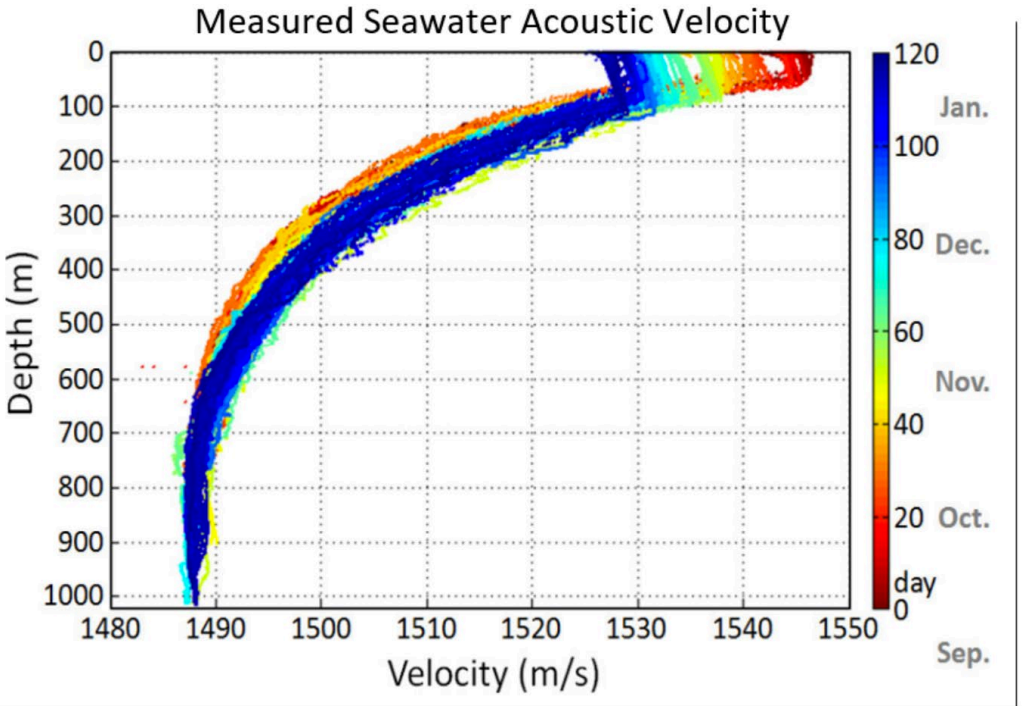
Example of data misfit: Arts et al, 2007 (Sleipner CO2 injection field)

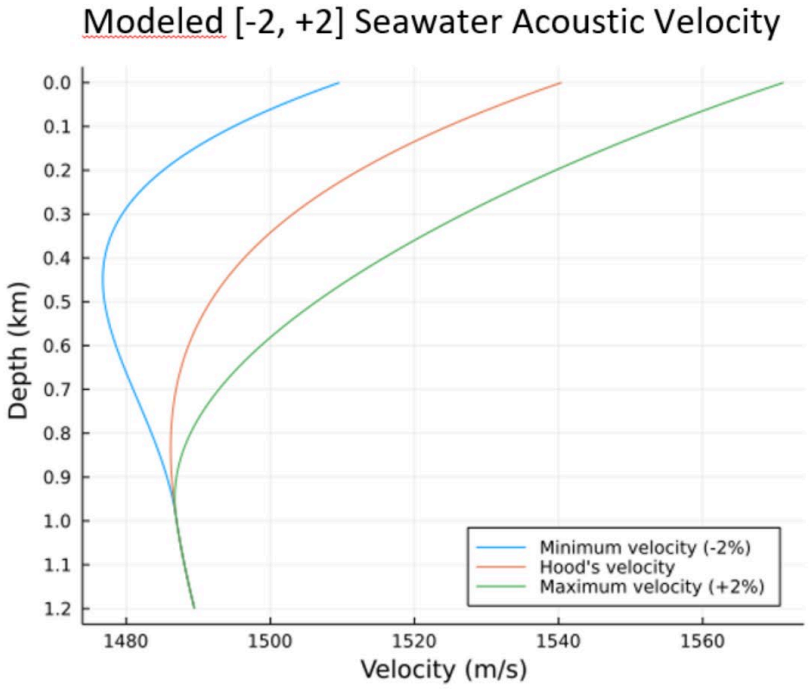
Co-train from simulations and data

Explaining beyond modeling

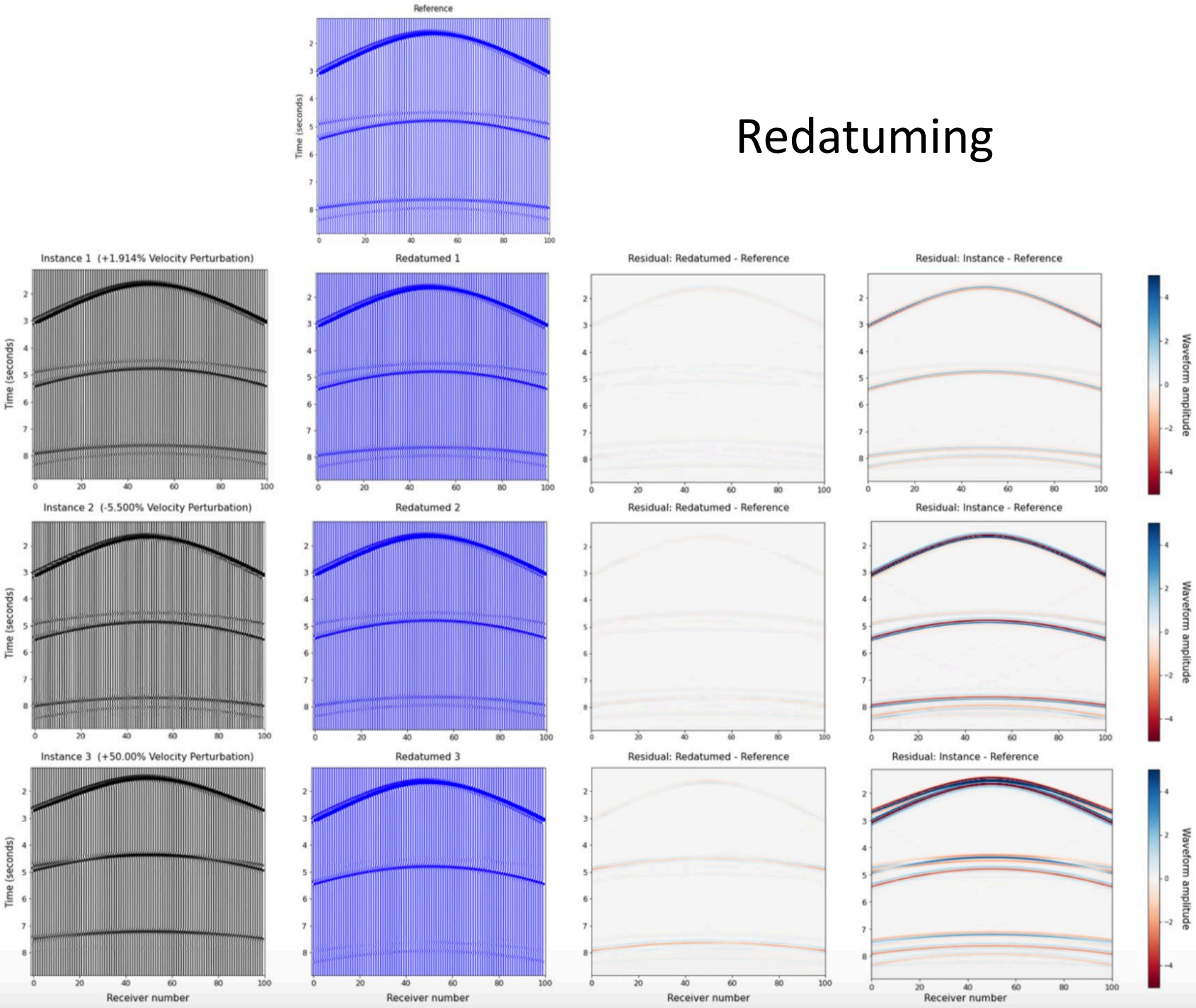


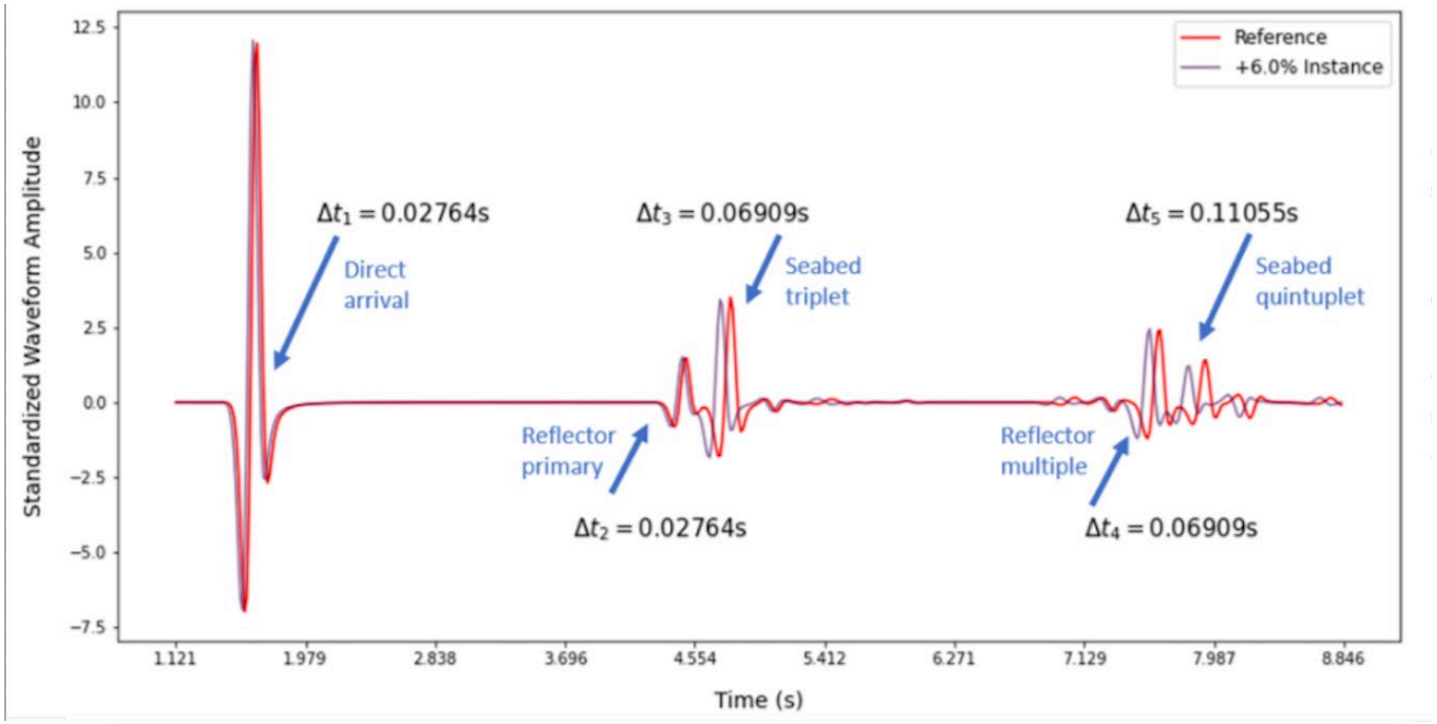
Example: Deepwater statics

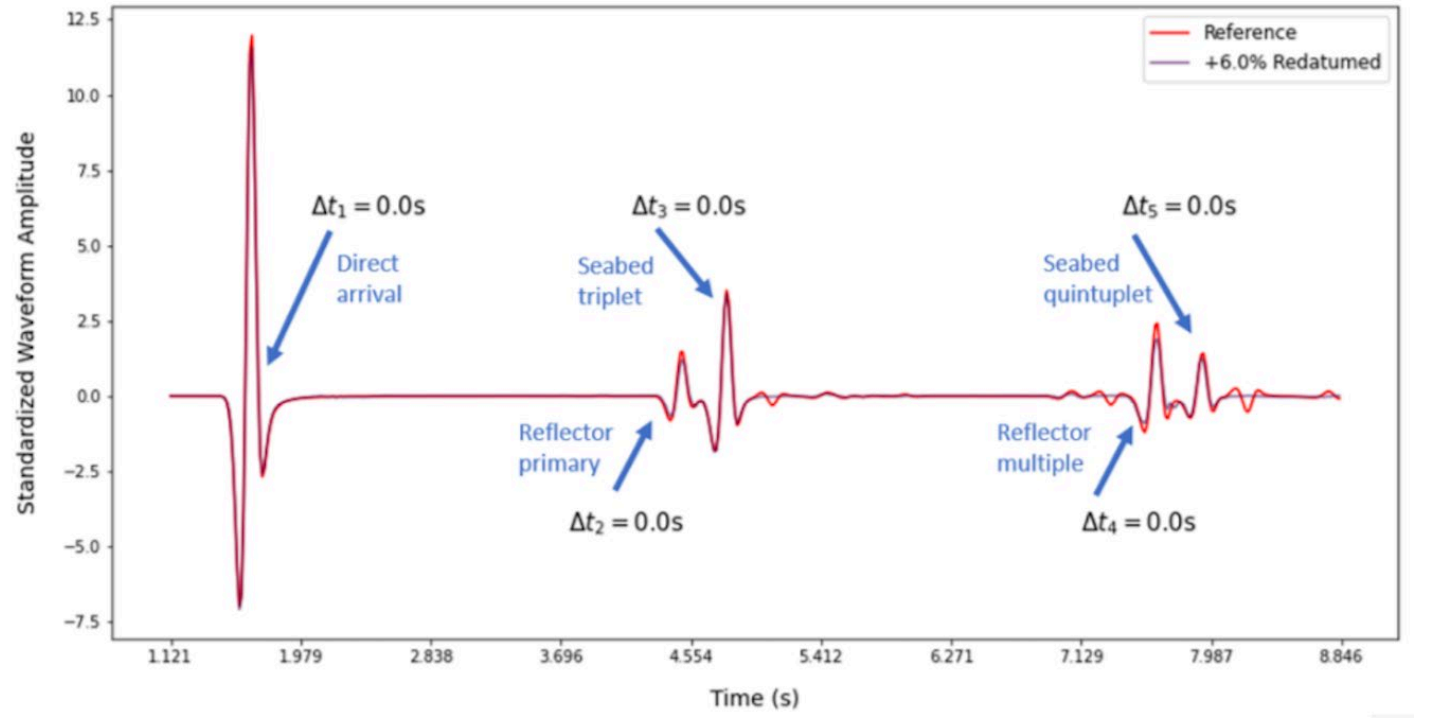




Redatuming


Reference
Instance 1 (+1.914% Velocity Perturbation)
Redatumed 1
Residual: Redatumed - Reference
Residual: Instance - Reference
Instance 2 (-5.500% Velocity Perturbation)
Redatumed 2
Residual: Redatumed - Reference
Residual: Instance - Reference
Instance 3 (+50.00% Velocity Perturbation)
Redatumed 3
Residual: Redatumed - Reference
Residual: Instance - Reference
Time (seconds)
Receiver number
Waveform amplitude


Reference
+6.0% Instance
Standardized Waveform Amplitude
Time (s)
$\Delta t_1 = 0.02764$s
Direct arrival
$\Delta t_3 = 0.06909$s
Seabed triplet
$\Delta t_5 = 0.11055$s
Seabed quintuplet
Reflector primary
$\Delta t_2 = 0.02764$s
Reflector multiple
$\Delta t_4 = 0.06909$s


Reference
+6.0% Redatumed
Standardized Waveform Amplitude
Time (s)
$\Delta t_1 = 0.0$s
Direct arrival
$\Delta t_3 = 0.0$s
Seabed triplet
$\Delta t_5 = 0.0$s
Seabed quintuplet
Reflector primary
$\Delta t_2 = 0.0$s
Reflector multiple
$\Delta t_4 = 0.0$s