“Modeling climate change: A dynamical systems approach”

Alejandro Aceves

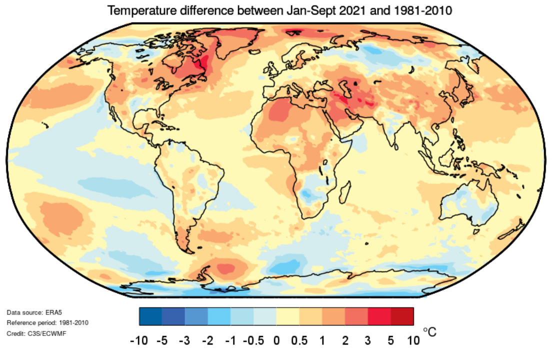
Department of Mathematics, SMU

FAU DCN-AvH Seminar

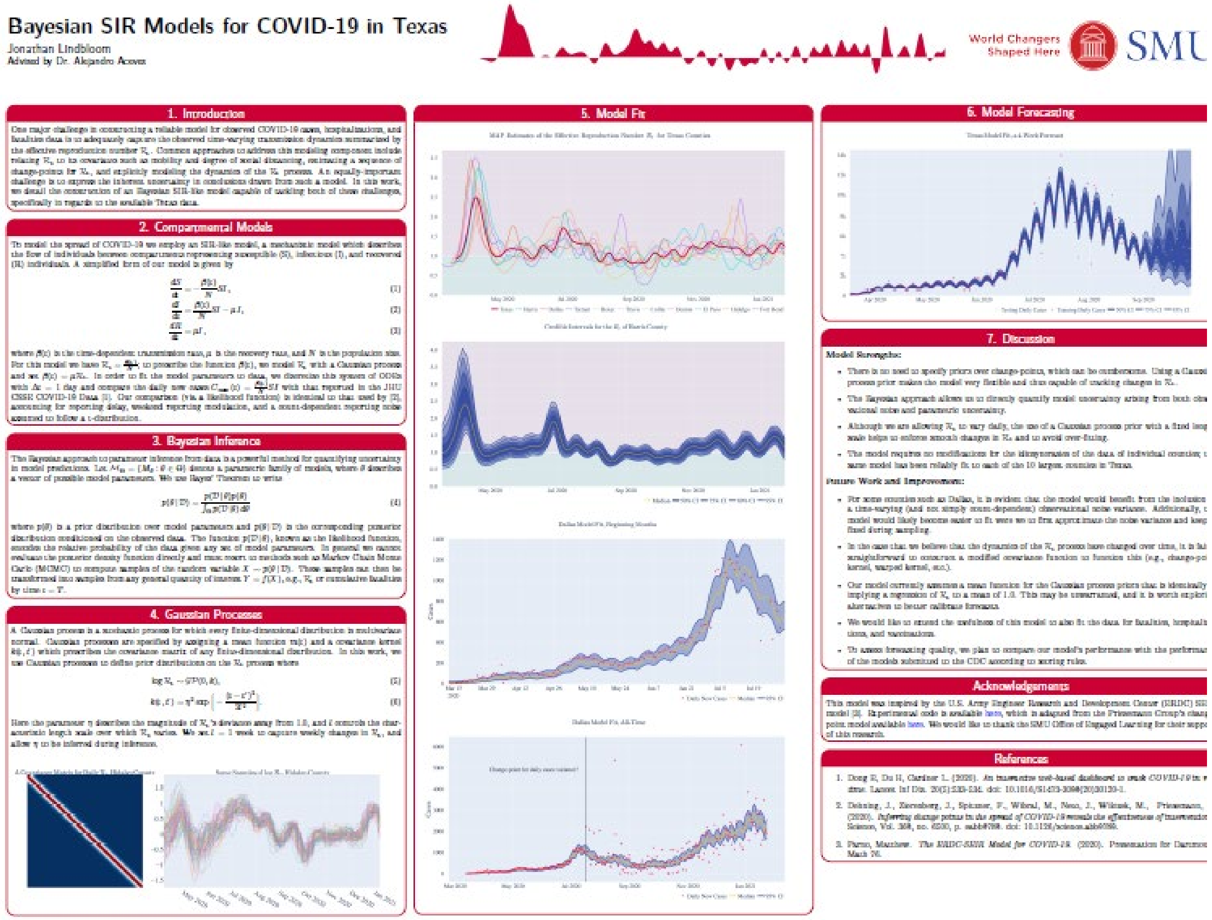
November 10, 2021

Mathematics and Climate Research Network

www.mathclimate.org



While we were “grounded” (lessons learned)



Research Article

Comparison of Predictive Models and Impact Assessment of Lockdown for COVID-19 over the United States

Olusola S. Makinde[1], Abiodun M. Adeola[2,3,*], Gbenga J. Abiodun[4], Olubukola O. Olusola-Makinde[5], Aceves Alejandro[4]

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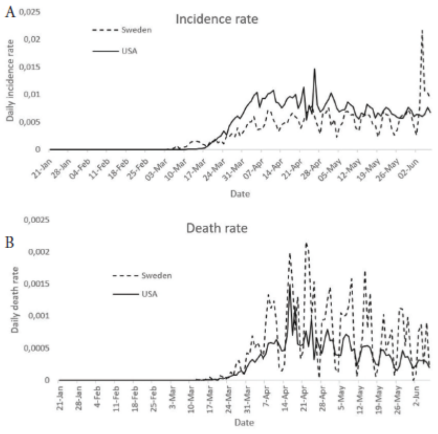


Figure 8 Comparison of incidence rates and death rates in the USA and Sweden, accounting for efficiency of lockdown policy.

Q: covid-19 research posing the question, were there early warning signs (EWS) ?

The 2021 Nobel Prize in Physics

- ***Syukuro Manabe**, Princeton University, and **Klaus Hasselmann,** Max Planck Institute for Meteorology, Hamburg, Germany, "for the physical modelling of Earth's climate, quantifying variability and reliably predicting global warming," and **Giorgio Parisi,** Sapienza University of Rome, Italy, "for the discovery of the interplay of disorder and fluctuations in physical systems from atomic to planetary scales."*

SMU

Syukuro Manabe
("I'm just a climatologist")

The Effects of Doubling the CO_2 Concentration on the Climate of a General Circulation Model[1]

SYUKURO MANABE AND RICHARD T. WETHERALD

Geophysical Fluid Dynamics Laboratory/NOAA, Princeton University, Princeton, N.J. 08540

(Manuscript received 6 June 1974, in revised form 8 August 1974)

ABSTRACT

An attempt is made to estimate the temperature changes resulting from doubling the present CO_2 concentration by the use of a simplified three-dimensional general circulation model. This model contains the following simplifications: a limited computational domain, an idealized topography, no heat transport by ocean currents, and fixed cloudiness. Despite these limitations, the results from this computation yield some indication of how the increase of CO_2 concentration may affect the distribution of temperature in the atmosphere. It is shown that the CO_2 increase raises the temperature of the model troposphere, whereas it lowers that of the model stratosphere. The tropospheric warming is somewhat larger than that expected

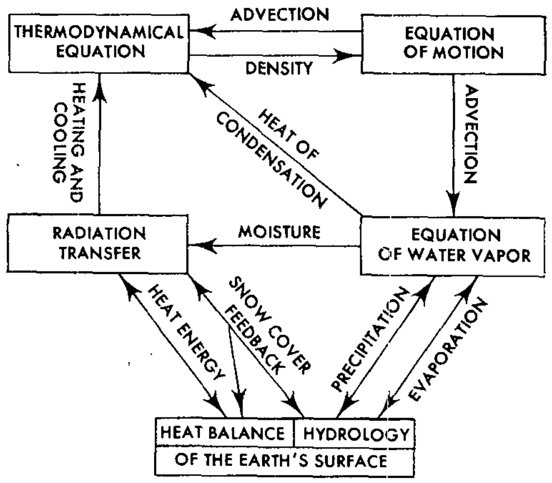


FIG. 2. Box diagram indicating the major components of the model. Arrows represent the links between components.

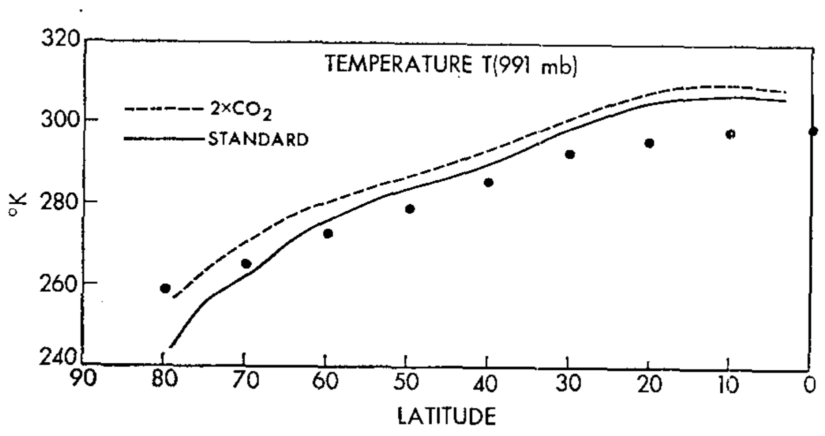
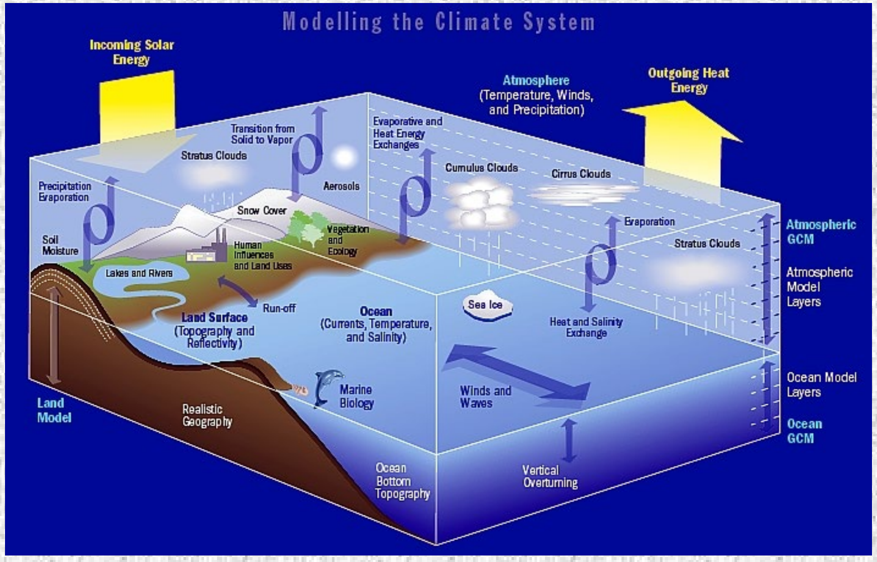


FIG. 5. Zonal mean temperature at the lowest prognostic level (i.e., ~991 mb). Dots indicate the observed distribution of zonal mean surface air temperature (Oort and Rasmusson, 1971).



Klaus Hasselmann
Oceanographer, father of Kinetic Equation (WT)

On the non-linear energy transfer in a gravity-wave spectrum

Part 1. General theory

By K. HASSELMANN
Institute for Naval Architecture, University of Hamburg†

On the non-linear energy transfer in a gravity wave spectrum

Part 2. Conservation theorems; wave-particle analogy; irreversibility

By K. HASSELMANN
Institute of Geophysics and Planetary Physics and Department of Oceanography, University of California, La Jolla

(Received 7 August 1962)

On the non-linear energy transfer in a gravity-wave spectrum

Part 3. Evaluation of the energy flux and swell-sea interaction for a Neumann spectrum

By K. HASSELMANN
Institute of Geophysics and Planetary Physics and Department of Oceanography, University of California, La Jolla

(Received 7 August 1962)

Klaus Hasselmann
Oceanographer, father of Kinetic Equation (WT)

REVIEWS OF GEOPHYSICS

VOLUME 4 FEBRUARY 1966 NUMBER 1

Feynman Diagrams and Interaction Rules of Wave-Wave Scattering Processes

K. HASSELMANN

Institute of Geophysics and Planetary Physics,
University of California, San Diego
Institute of Naval Architecture, University of Hamburg

Abstract. The energy transfer due to weak nonlinear interactions in random wave fields is reinterpreted in terms of a hypothetical ensemble of interacting particles, antiparticles, and virtual particles. In the particle picture, the interactions can be conveniently described by Feynman diagrams, which may be regarded either as branch diagrams of the perturbation expansion or as collision diagrams. The derivation of the transfer expressions can then be reduced to a few general rules for the construction of the diagrams and the associated collision cross sections. The representation follows closely the standard treatment of nonlinear lattice vibrations, but the particle picture differs from the usual phonon interpretation of lattice waves. It has the unrealistic property that the energies and number densities of antiparticles are negative. This is offset by simpler interaction rules and a closer correspondence between the perturbation graphs and collision diagrams. The method is illustrated for scattering processes in the oceanic wave guide involving surface and internal gravity waves, horizontal currents (turbulence), seismic waves, and bottom irregularities.

$$
\begin{aligned}
\frac{\partial n_\nu(\mathbf{k})}{\partial t} = & \sum_{\nu_1,\nu_2>0} \int \cdots \int d\mathbf{k}_1\, d\mathbf{k}_2 \\
& \cdot \{ T^{\nu_1 \nu_2 -\nu}_{\mathbf{k}_1 \mathbf{k}_2 -\mathbf{k}} (n_1 n_2 - n n_1 - n n_2) + 2 T^{\nu_1 -\nu_2 -\nu}_{\mathbf{k}_1 -\mathbf{k}_2 -\mathbf{k}} (n_1 n_2 + n n_1 + n n_2) \} \\
& + \sum_{\nu_1,\nu_2,\nu_3>0} \int \cdots \int d\mathbf{k}_1\, d\mathbf{k}_2\, d\mathbf{k}_3 \{ T^{\nu_1 \nu_2 \nu_3 -\nu}_{\mathbf{k}_1 \mathbf{k}_2 \mathbf{k}_3 -\mathbf{k}} (n_1 n_2 n_3 - n n_2 n_3 - n n_1 n_3 - n n_1 n_2) \\
& + 3 T^{\nu_1 \nu_2 -\nu_3 -\nu}_{\mathbf{k}_1 \mathbf{k}_2 -\mathbf{k}_3 -\mathbf{k}} (n_1 n_2 n_3 - n n_2 n_3 - n n_1 n_3 + n n_1 n_2) \\
& + 3 T^{\nu_1 -\nu_2 -\nu_3 -\nu}_{\mathbf{k}_1 -\mathbf{k}_2 -\mathbf{k}_3 -\mathbf{k}} (n_1 n_2 n_3 - n n_2 n_3 + n n_1 n_3 + n n_1 n_2) \} + \sum_{\nu_1,\nu_2,\nu_3,\nu_4>0} \cdots \qquad (2.5)
\end{aligned}
$$

Giorgio Parisi

From Universita di Roma, Sapienza Webpage (shown here 2 stories)

Giorgio Parisi is Nobel Prize in Physics

A rainbow-like spiral to boost telecommunications

Tellus (1982) *34*, 10–16

Stochastic resonance in climatic change

By ROBERTO BENZI, *Istituto di Fisica dell'Atmosfera, C.N.R., Piazza Luigi Sturzo 31, 00144, Roma, Italy,*
GIORGIO PARISI, *I.N.F.N., Laboratori Nazionali di Frascati, Frascati, Roma, Italy,*
ALFONSO SUTERA, *The Center for the Environment and Man, Hartford, Connecticut 06120, U.S.A.*
and ANGELO VULPIANI, *Istituto di Fisica "G. Marconi", Università di Roma, Italy*

(Manuscript received November 12, 1980; in final form March 13, 1981)

ABSTRACT

An amplification of random perturbations by the interaction of non-linearities internal to the climatic system with external, orbital forcing is found. This stochastic resonance is investigated in a highly simplified, zero-dimensional climate model. It is conceivable that this new type of resonance might play a role in explaining the 10^5 year peak in the power spectra of paleoclimatic records.

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\varepsilon(T)}{C} \times \left\{ \frac{\mu(t)}{1 + \beta[(1 - T/T_1)(1 - T/T_2)(1 - T/T_3)} - 1 \right\}. \tag{4}$$

Here $T_1 < T_2 < T_3$ are the three hypothesized climates, and μ is given by

$\mu(t) = 1 + 0.0005 \cos \omega t$

where

$\omega = 2\pi/10^5$ years

$$\frac{dT}{dt} = \frac{\varepsilon(T)}{C} \times \left\{ \frac{\mu(t)}{1 + \beta[(1 - T/T_1)(1 - T/T_2)(1 - T/T_3)]} - 1 \right\}. \quad (4)$$

Here $T_1 < T_2 < T_3$ are the three hypothesized climates, and μ is given by

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where

$$\omega = 2\pi/10^5 \text{ years}$$

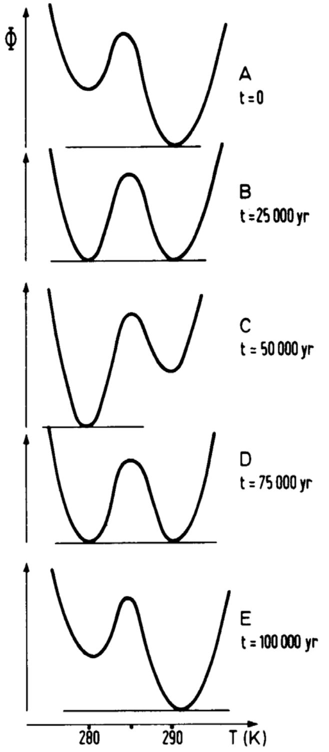


Fig. 3. The behaviour of the pseudo-potential $\Phi(T; t)$ as a function of time for a model of the Ghil–Bhattacharya (1979) type. Note that the maxima and minima of $\Phi(T; t)$ are changing only by a few tenths of degree during the cycle, while the pseudo-potential difference between the stable points and the unstable point is changing by a factor of two.

RESULTS:

Plus additive noise:

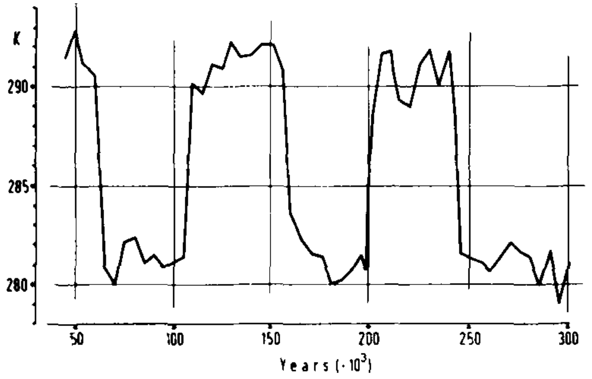


Fig. 4. Computer simulation of eq. (5) for heat-budget model with two observable climates at 280 and 290 K. The variance of the noise was about 0.15 K^2/year.

5. Conclusions

Our results point to the possibility of explaining large amplitude, long-term alternations of temperature by means of a co-operation between external periodic forcing due to orbital variations and an internal stochastic mechanism. The external periodic forcing alone is unable to reproduce the major peak in the observed quaternary climate records. The internal stochastic forcing alone does not reproduce it either. The combination of the two effects, however, produces what we may call a stochastic resonance, which amplifies the small external forcing: a small change in the external forcing induces a large change in the probability of jumping between two observable climates. This new mechanism could be useful in our understanding of long-term climatic change. At any rate, it seems to warrant further investigation.

Random "kicking" of a particle in a double well potential

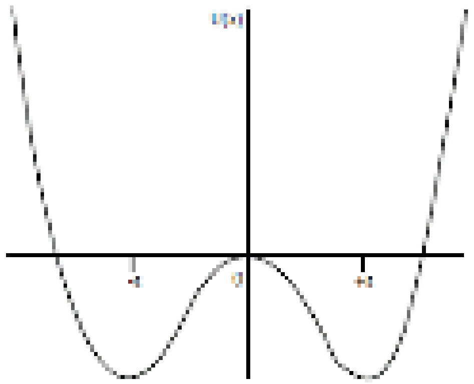
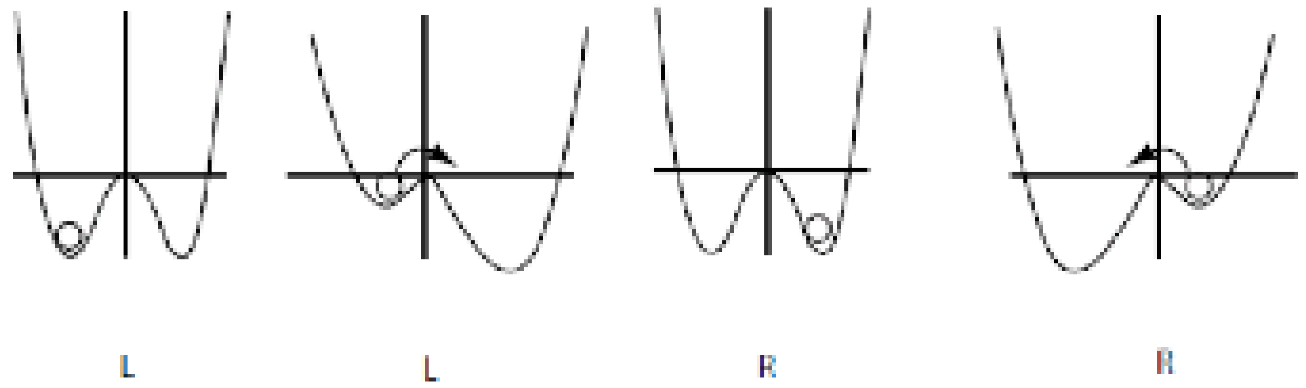


Fig. 3. (a) Bistable double well potential



Parisi published in SIAM

SIAM J. APPL. MATH
Vol. 43, No. 3, June 1983

0036–1399/83/4303–0009 $01.25/0

A THEORY OF STOCHASTIC RESONANCE IN CLIMATIC CHANGE*

ROBERTO BENZI,† GIORGIO PARISI,‡ ALFONSO SUTERA§ AND ANGELO VULPIANI¶

Abstract. In this paper we study a one-dimensional, nonlinear stochastic differential equation when small amplitude, long-period forcing is applied. The equation arises in the theory of the climate of the earth. We find that the cooperative effect of the stochastic perturbation and periodic forcing lead to an amplification of the peak of the power spectrum, due to a mechanism that we call stochastic resonance. A heuristic analysis of the resonance condition is presented and our analytical findings are confirmed by numerical calculations.

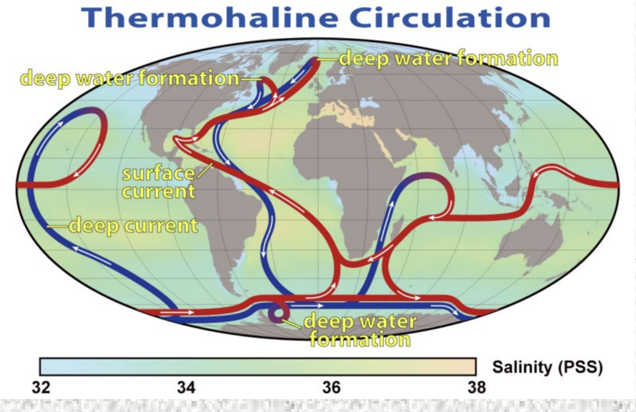
Atlantic meridional overturning circulation- A dynamical systems approach

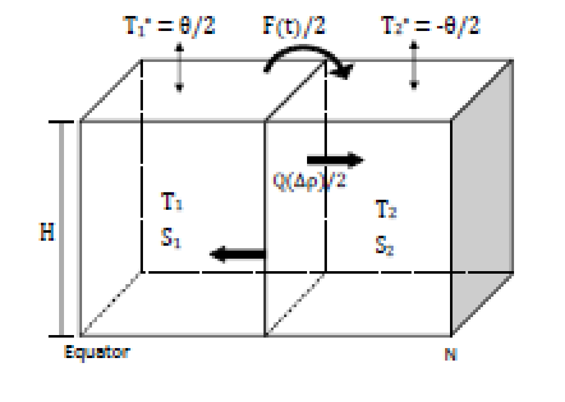
"Predicting dynamic trends of the Atlantic meridional overturning circulation for transient and stochastic forcing effects"
Alyssa Pampell, A. A, and Gowri Srinivasan. SIAM/ASA Journal on Uncertainty Quantification V. 2(1), pp. 585-606 (2014)

Our work investigates two important factors that can alter AMOC dynamics: temperature variability and stochastic freshwater variations. These two factors in combination can lead to undesired transitions to the weak state, resulting in abrupt cooling and a reduction in precipitation in the North Atlantic and Europe [S. Rahmstorf and A. Ganopolski, Climatic Change, 43 (1999), pp. 353–367].

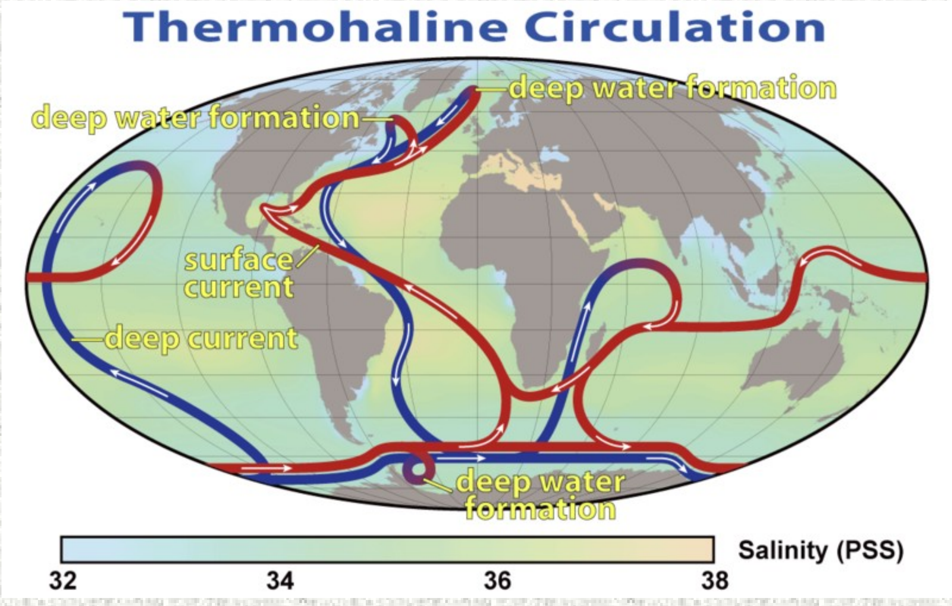
Alyssa Pampell Manis, Orbital Debris Scientist and Project Manager at HX5, LLC (NASA)

Gowri Srinivasan
Deputy Group Leader, Physics Division LANL





Ocean Circulation



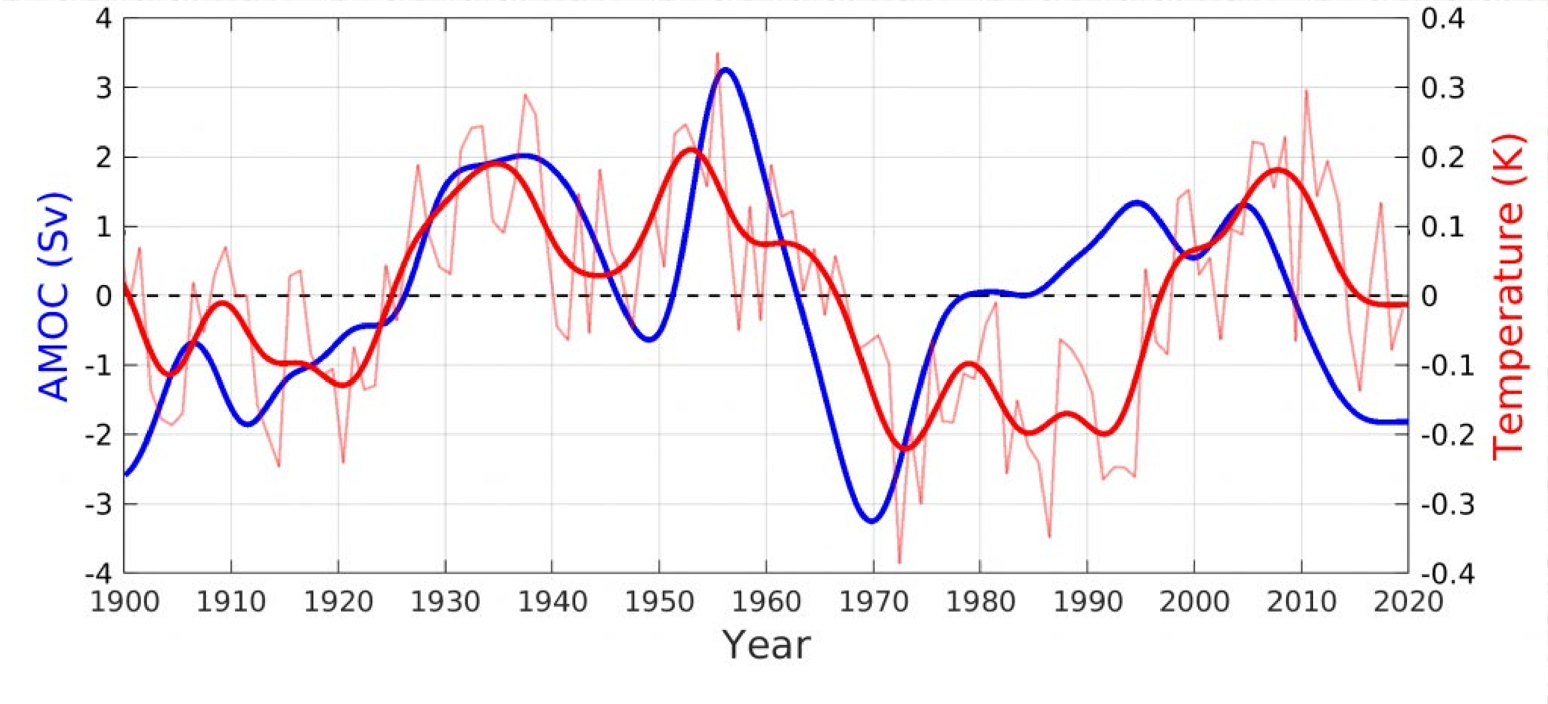
http://en.wikipedia.org/wiki/Thermohaline_circulation

- Climate patterns largely influenced by oceanic circulation
  - Earth is 71% ocean
  - Large oceanic heat capacity buffers climate change
  - Transports heat, salt, affects biological systems
- Of particular interest is Atlantic meridional overturning circulation (AMOC) - measure of Atlantic flow across latitudes

Why study AMOC

From news.arizona.edu (October 2021): "This (AMOC) circulation transports an enormous amount of heat northward in the ocean," Yin said. "The magnitude is on the order of 1 petawatts, or 10 to the 15 power watts. Right now, the energy consumption by the entire world is about 20 terawatts, or 10 to the 12 power watts. So, 1 petawatt is enough to run about 50 civilizations."

The AMOC is commonly defined at a given latitude using a streamfunction Ψ in units of Sverdrups (1 Sv = 10^6 m^3/s)—the zonally-integrated and vertically-accumulated meridional volume transport in depth coordinates.



Geophysical Researcg Letters, "120 Years of AMOC Variability Reconstructed From Observations Using the Bernoulli Inverse

Neil J. Fraser, Stuart A. Cunningham , 07 September 2021

A dynamical systems approach: Tipping points

- **Bifurcation-induced tipping**[edit]
- This occurs when a particular parameter in the climate, which is observed to be consistently moving in a given direction over a period of time, eventually passes through a critical level - at which point a dangerous bifurcation, or fork takes place - and what was a stable state loses its stability or simply disappears.[27] The Atlantic Meridional Overturning Circulation (AMOC) is like a conveyor belt driven by thermohaline circulation. Slow changes to the bifurcation parameters in this system — the salinity, temperature and density of the water - have caused circulation to slow down by about 15% in the last 70 years or so. If it reaches a critical point where it stops completely, this would be an example of bifurcation induced tipping.[28][29]
- **Noise-induced tipping**[edit]
- This refers to transitions from one state to another due to random fluctuations or internal variability of the system. Noise-induced transitions show none of the early warning signals which occur with bifurcations. This means they are fundamentally unpredictable as there is no systematic change in the underlying parameters. Because they are unpredictable, such occurrences are often described as a ‘one-in-x-year’ event.[30] An example is the Dansgaard–Oeschger events during the last glacial period, with 25 occurrences of sudden climate fluctuations over a 500 year period.[31]
- **Rate-induced tipping**[edit]
- This aspect of tipping assumes that there is a unique, stable state for any fixed aspect or parameter of the climate and that, if left undisturbed, there will only be small responses to a ‘small’ stimulus. However, when changes in one of the system parameters begin to occur more rapidly, a very large 'excitable' response may appear. In the case of peatlands, for instance, after years of relative stability, the rate-induced tipping point leads to an "explosive release of soil carbon from peatlands into the atmosphere" - sometimes known as "compost bomb instability

Atlantic Meridional Overturning Circulation (AMOC)

- Flow controlled largely by density differences (due to temperature and salinity gradients)
- Northward surface flow from equator to pole, deep return flow
- Surface flow transports heat and salt from tropics northward
- Heat transport strongly affects climate in North Atlantic
  - Sea surface temps. up to about 5° C warmer in North Atlantic compared to similar latitudes of north Pacific
- Density increase along flow due to evaporation (increased salinity) and cooling
- Overturning circulation: sinking of dense water in high northern latitudes
  - Only region in northern hemisphere where deep water forms

AMOC and climate change

- Effect on North Atlantic climate & ocean circulation patterns makes AMOC important factor in climate studies
- Past abrupt climate change linked to weakening of AMOC
- Studies show AMOC strength is sensitive to surface freshwater fluxes
  - General hydrological cycle changes
  - Melting of sea ice/glaciers
- Feedback between climate change and AMOC: increased freshwater or surface temperatures due to climate change $\rightarrow$ weakening of AMOC $\rightarrow$ further climate changes in North Atlantic (and elsewhere)

Modeling Ocean Circulation

Ocean circulation governed by full geophysical Navier Stokes equations + scalar PDEs for temperature, salinity

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -g\rho \boldsymbol{e}_z - \frac{1}{\rho}\nabla p - 2(\boldsymbol{\Omega} \times \boldsymbol{u}) + \nu \nabla^2 \boldsymbol{u}$$

$$\nabla \cdot \boldsymbol{u} = 0$$

$$\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T = \kappa_T \nabla^2 T$$

$$\frac{\partial S}{\partial t} + \boldsymbol{u} \cdot \nabla S = \kappa_S \nabla^2 S$$

$\boldsymbol{u} = [u, v, w]^T$ = velocity
g = gravitational acceleration
ρ = density
p = pressure

ν = viscosity (eddy diffusivity)
κ_T, κ_S = thermal, haline conductivities (eddy diffusivity)

$-2(\boldsymbol{\Omega} \times \boldsymbol{u})$ = Coriolis acceleration, $\Omega = |\boldsymbol{\Omega}| = 7.29 \times 10^{-5}$ rad s^{-1}.

Simplifications

- Neglect external forces (Coriolis effect), average over longitudes → 2D system
- Introduce stream function ψ where $v = -\partial_z \psi$ and $w = \partial_y \psi$
- Gravity → bouyancy term using a linear equation of state: $-g\rho = g(\alpha_T T - \alpha_S S)$, where α_T and α_S are the thermal, haline expansion coefficients

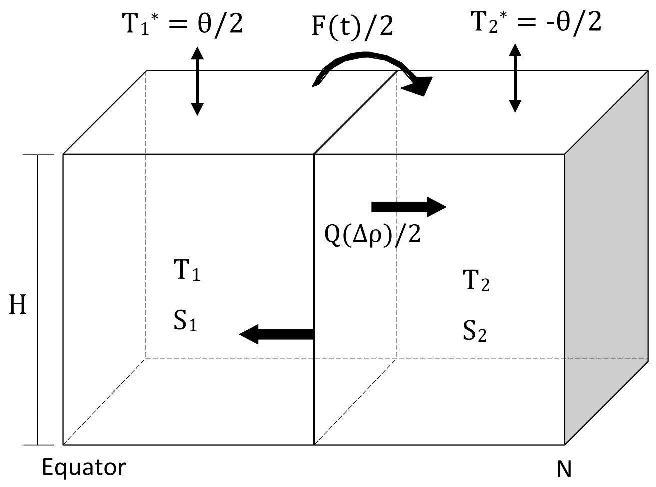
$$
\begin{aligned}
\partial_t \nabla^2 \psi + J(\psi, \nabla^2 \psi) &= g(\alpha_T T_y - \alpha_S S_y) + \nu \nabla^4 \psi \\
\partial_t T + J(\psi, T) &= \kappa_T \nabla^2 T \\
\partial_t S + J(\psi, S) &= \kappa_S \nabla^2 S
\end{aligned}
$$

with the Jacobian operator $J(A, B) = A_y B_z - A_z B_y$

Box models

- Neglect viscosity, average over spatial regions $\rightarrow$ system of ODEs
- Simplified model called a "box model"
  - Discretizes ocean as small number of spatially discrete boxes that are homogeneously mixed
  - May couple other processes via fluxes
  - Computationally very efficient
  - Allow for conceptualization of causal relationships between different components of ocean circulation (temperature, salinity, etc.)
  - Can inform relationships seen in large, numerical models that are not always obvious due to complexity
  - "Without exception, all big ideas have come from simple models" - Ray Pierrhumbert, climate scientist at U Chicago

Two-box model



- Framework for AMOC in single hemisphere, based on [1]
- Box 1: low-latitude Atlantic, Box 2: high-latitude North Atlantic
- T = temperature, S = salinity
- Symmetric restoring temperature conditions $\pm\theta/2$
- Evaporation − precipitation flux $F(t)/2$ at surface, transports freshwater from box 1 to box 2

[1] Cessi, P. A simple box model of stochastically forced thermohaline flow. *J. Phys. Oceanogr.* *24* (1994), 1911-1920.

Dynamical system

2-box model cont.

- Equation of state for ocean:

$$\rho = \rho_0 \left[1 - \alpha_T(T - T_0) + \alpha_S(S - S_0)\right]$$

- Conservation equations:

$$\dot{T}_1 = -t_r^{-1}\left(T_1 - \frac{\theta}{2}\right) - \frac{1}{2}Q(\Delta\rho)(T_1 - T_2)$$

$$\dot{T}_2 = -t_r^{-1}\left(T_2 + \frac{\theta}{2}\right) - \frac{1}{2}Q(\Delta\rho)(T_2 - T_1)$$

$$\dot{S}_1 = \frac{F(t)}{2H}S_0 - \frac{1}{2}Q(\Delta\rho)(S_1 - S_2)$$

$$\dot{S}_2 = -\frac{F(t)}{2H}S_0 - \frac{1}{2}Q(\Delta\rho)(S_2 - S_1)$$

ρ= density

$\alpha_T = -\frac{1}{\rho_0}\frac{\partial\rho}{\partial T}$= thermal expansion coefficient

$\alpha_S = \frac{1}{\rho_0}\frac{\partial\rho}{\partial S}$= salinity coefficient

t_r = temperature restoring time

H = depth of model ocean

Model we studied

A. Pampell, A.A, G. Srinivasan, SIAM/ASA J. Uncertainty Quantification, 2(1), 585–606. (2014)

- Define gradients $\Delta T = T_1 - T_2$, $\Delta S = S_1 - S_2$
- Nondimensionalize:

$$x = \frac{\Delta T}{\theta_0} \qquad t' = t_d^{-1} t \qquad \mu^2 = \frac{t_d q (\alpha_T \theta)^2}{V}$$

$$y = \frac{\alpha_S \Delta S}{\alpha_T \theta_0} \qquad \alpha = \frac{t_d}{t_r}$$

$\Rightarrow$

$$\dot{x} = -\alpha(x - \theta(\tau)) - x[1 + \mu^2 (x - y)^2]$$
$$\dot{y} = p - y[1 + \mu^2 (x - y)^2]$$

- Typical timescales $t_r = 25$ days, $t_d = 219$ years $\Rightarrow \alpha \approx 3.2 \times 10^3$
- Introduce $\epsilon = 1/\alpha \approx 3.1 \times 10^{-4}$

$$0 = x - \theta(\tau) + \mathcal{O}(\epsilon)$$
$$\dot{y} = p - y[1 + \mu^2 (\theta(\tau) - y)^2] + \mathcal{O}(\epsilon)$$

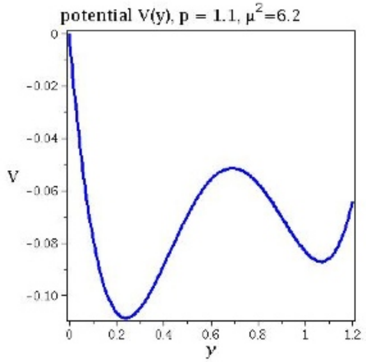
- Can approximate $x \approx \theta(\tau) \Rightarrow$ one ODE for y

Role of freshwater in potential configuration

3 equilibria

- Equation for y corresponds to a trajectory in a double-welled potential

$$V(y) = m\left(\frac{y^4}{4} - \frac{2}{3}y^3 + \frac{y^2}{2}\right) + \frac{y^2}{2} - py$$



- $y_a \approx 0.24$, $y_b \approx 0.69$, $y_c \approx 1.07$

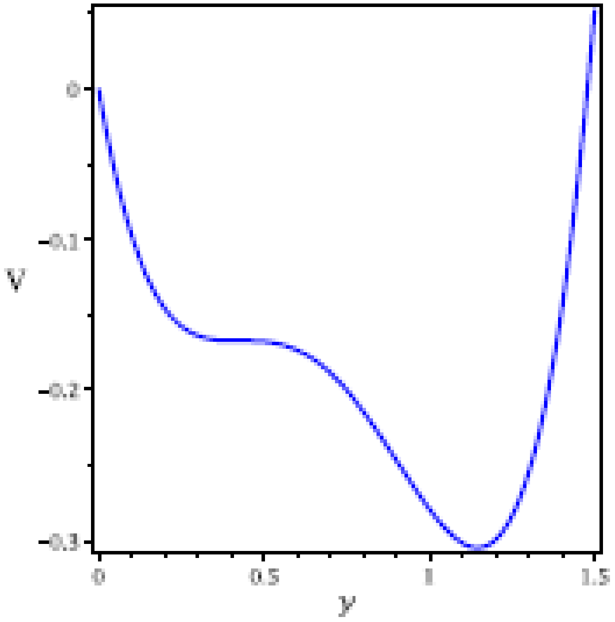
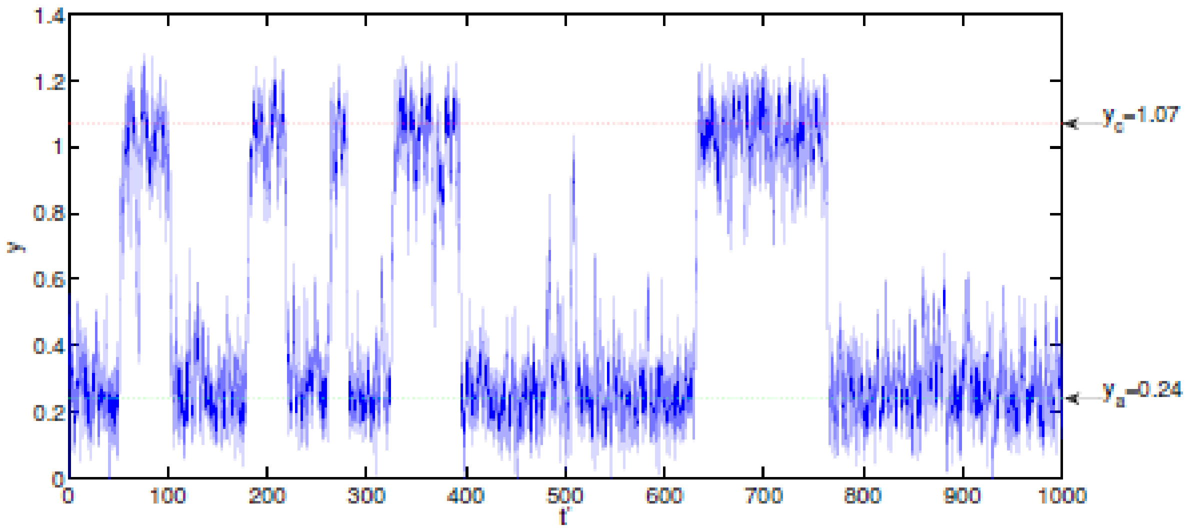


Figure: $p' = 0.2$

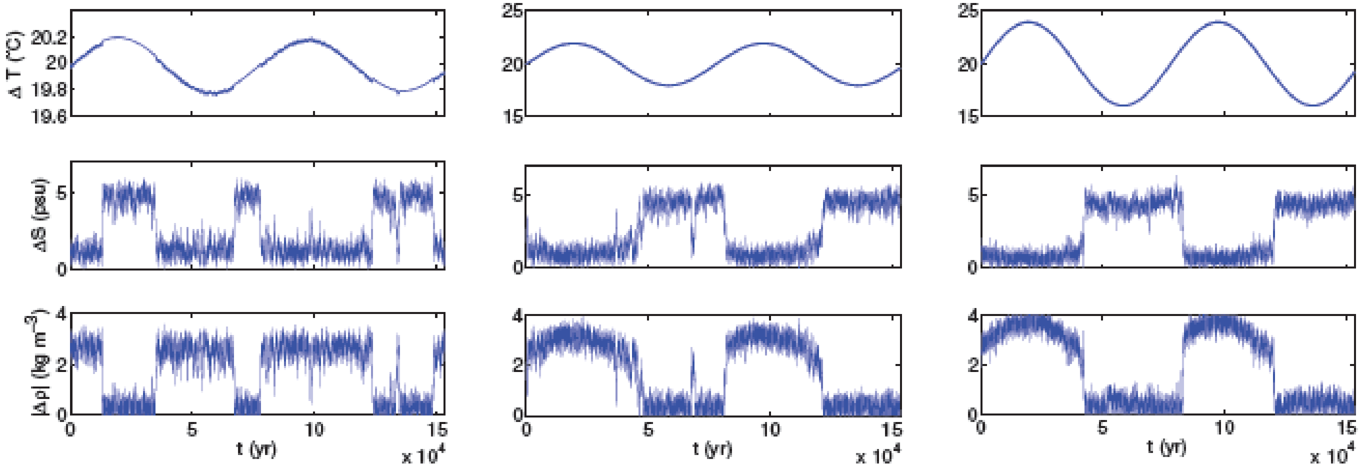
Sample time series solution for y with stochastic freshwater flux and constant restoring temperature



Build-up from simplest model

Ex: Add deterministic + stochastic forcing and mean temperature to 2 Box models

Results, Stochastic Perturbations



(a) $k = 0.01$ (b) $k = 0.1$ (c) $k = 0.2$

Figure: $\theta_2(\tau_2) = 1 + k \sin(\tau_2)$, stochastic

3-Box Model

A key feature of the AMOC is the cross-hemispheric pole-to-pole circulation $\Rightarrow$ need a model that captures expanse of Atlantic from Southern Ocean to North Atlantic

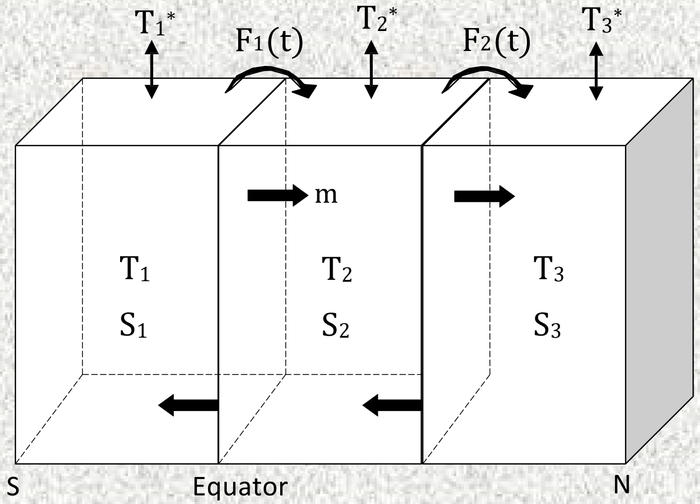


Figure: 3-box model

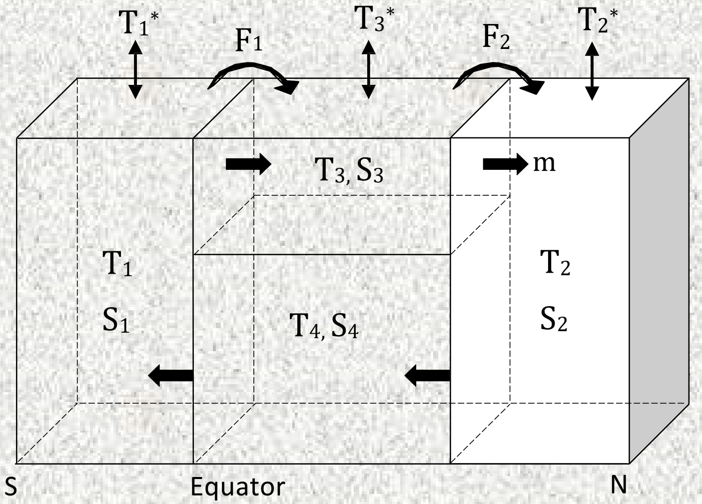


Figure: 4-box model

3-Box Equations

$$
\begin{aligned}
\dot{T}_1 &= \frac{m}{V_1}(T_2 - T_1) + \lambda_1(T_1^* - T_1) \\
\dot{T}_2 &= \frac{m}{V_2}(T_1 - 2T_2 + T_3) + \lambda_2(T_2^* - T_2) \\
\dot{T}_3 &= \frac{m}{V_3}(T_2 - T_3) + \lambda_3(T_3^* - T_3) \\
\dot{S}_1 &= \frac{m}{V_1}(S_2 - S_1) + \frac{F_1 S_0}{V_1} \\
\dot{S}_2 &= \frac{m}{V_2}(S_1 - 2S_2 + S_3) - \frac{(F_1 - F_2)S_0}{V_2} \\
\dot{S}_3 &= \frac{m}{V_3}(S_2 - S_3) - \frac{F_2 S_0}{V_3}.
\end{aligned}
$$

with $m = \frac{k\Delta\rho}{\rho_0} = k(\beta(S_3 - S_1) - \alpha(T_3 - T_1))$, in Sv (1 Sv=$10^6\,\mathrm{m}^3\,\mathrm{s}^{-1}$)

Here, transport strength $m \propto \Delta\rho$ (two-box had quadratic relationship)

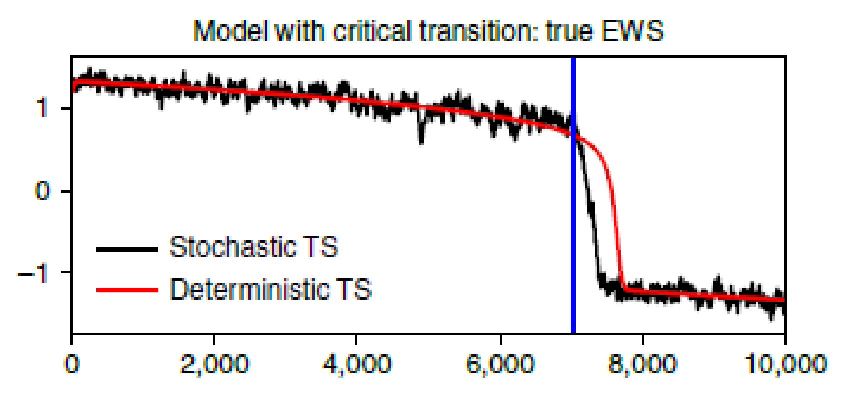
Observation-based early-warning signals for a collapse of the Atlantic Meridional Overturning Circulation

Niklas Boers

Nature Climate Change VOL 11 680, August 2021, 680–688 www.nature.com/natureclimatechange

The Atlantic Meridional Overturning Circulation (AMOC), a major ocean current system transporting warm surface waters toward the northern Atlantic, has been suggested to exhibit two distinct modes of operation. **A collapse from the currently attained strong to the weak mode would have severe impacts on the global climate system and further multi-stable Earth system components**. **Observations and recently suggested fingerprints of AMOC variability indicate a gradual weakening during the last decades, but estimates of the *critical transition point remain uncertain*.** Here, **a robust and general early-warning indicator for forthcoming critical transitions is introduced**. Significant early-warning signals are found in eight independent AMOC indices, **based on observational sea-surface temperature and salinity data from across the Atlantic Ocean basin**. These results reveal spatially consistent empirical evidence that, in the course of the last century, the AMOC may have evolved from relatively stable conditions to a point close to a critical transition.

Learning from simple models:



Model suggest the possibility of having early warning signals EWS.

$$\frac{dX}{dt} = -x^3 + x - T + \tilde{\eta}(t)$$

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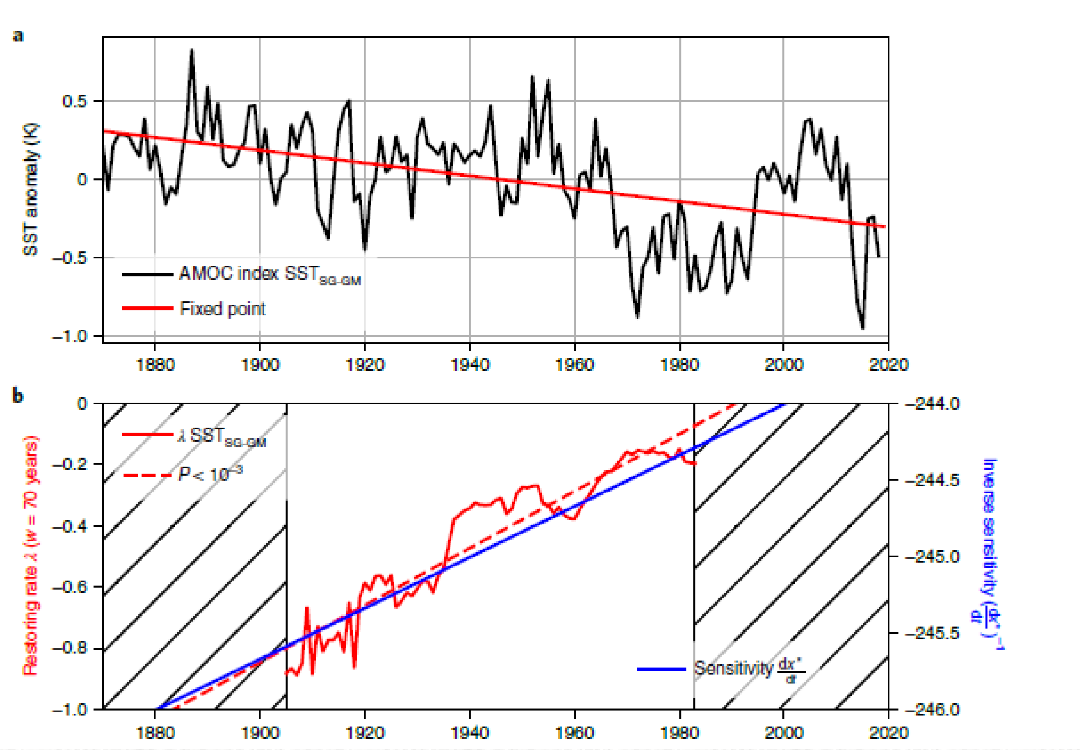
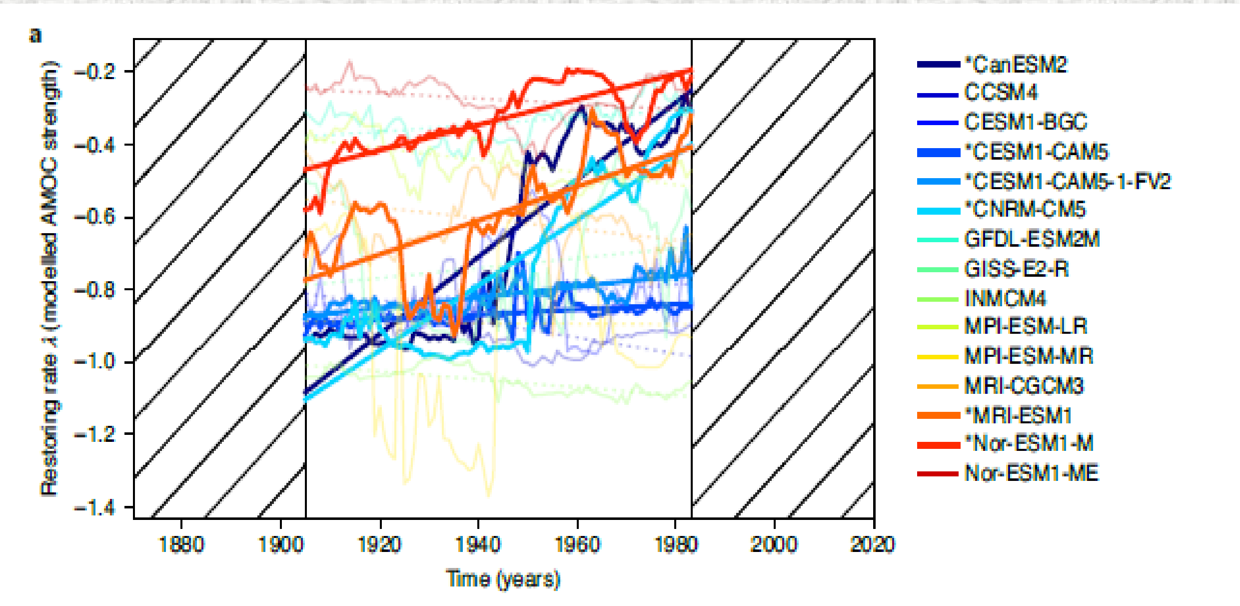
Close to an equilibrium point at a bifurcation: $\frac{dx}{dt} \approx \lambda x + \tilde{\eta}(t)$





Other recent work

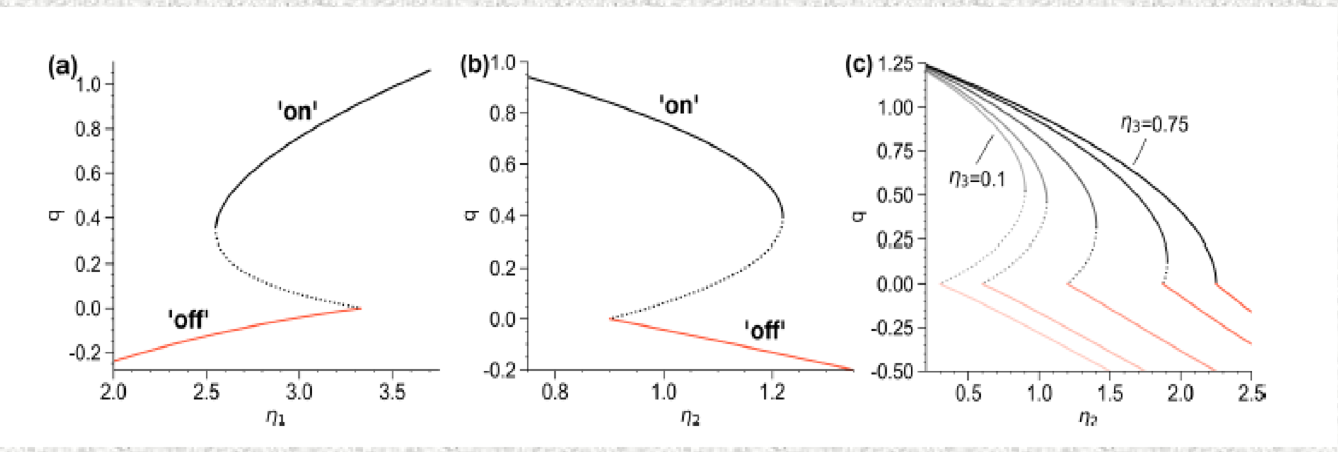
“Abrupt climate change as a rate-dependent cascading tipping point”

Johannes Lohmann, Daniele Castellana, Peter D. Ditlevsen, and Henk A. Dijkstra

Published: 28 July 2021, Earth Syst. Dynam., 12, 819–835, 2021

https://doi.org/10.5194/esd-12-819-2021

$$dT_t = (\eta_1 - T - |T - S|T)\,dt + \sigma_T\,dW_{T,t}$$
$$dS_t = (\eta_2 - \eta_3 S - |T - S|S)\,dt + \sigma_S\,dW_{S,t},$$



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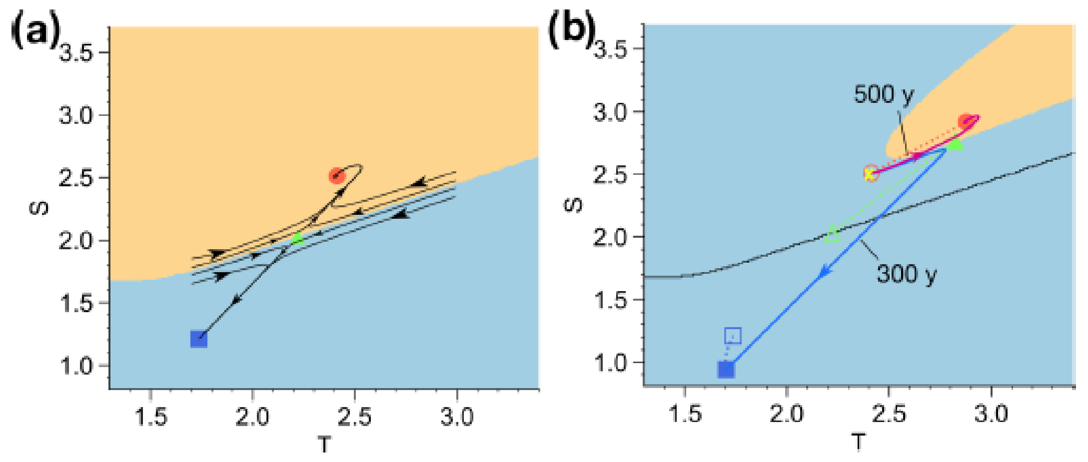
$$\mathrm{d}I_t = \left(\Delta \tanh\left(\frac{I}{h}\right) + [R_0\Theta(I) - B]I + L - F - 1 + R\right)\mathrm{d}t + \sigma_I \mathrm{d}W_{I,t}, \quad (7)$$

$$\frac{\tau_T}{\tau_I}\mathrm{d}T_t = \left(\eta_1^0 - \kappa\Theta(I)\cdot I - T - |T-S|T\right)\mathrm{d}t + \sigma_T \mathrm{d}W_{T,t}, \quad (8)$$

$$\frac{\tau_T}{\tau_I}\mathrm{d}S_t = (\eta_2 - \eta_3 S - |T-S|S)\mathrm{d}t + \sigma_S \mathrm{d}W_{S,t}. \quad (9)$$

Insert sea-ice component

DETERMINISTIC CASE



Rate induced tipping

Other recent work

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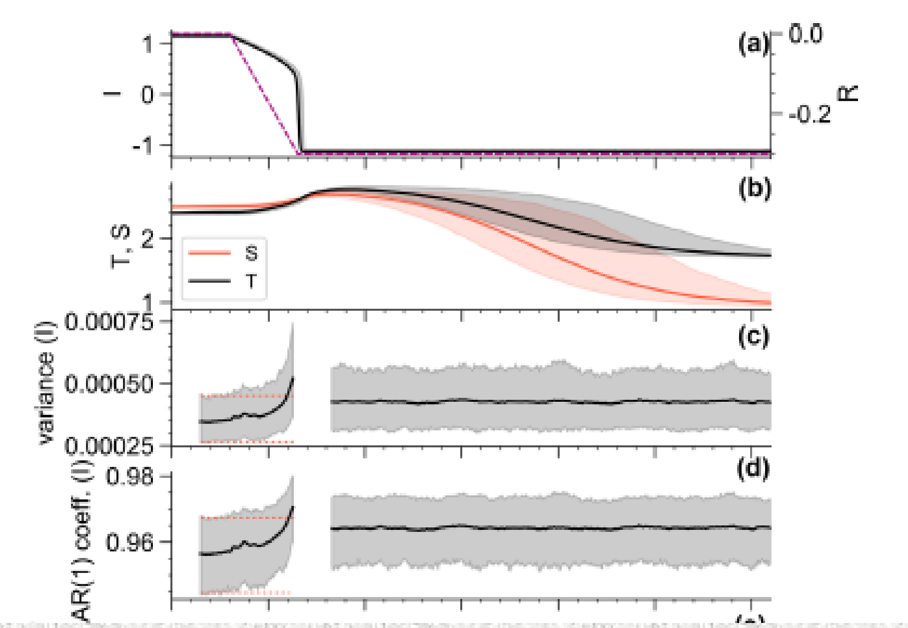
Early warning of the tipping cascade

Due to their irreversible nature, it is important to foresee impending tipping points using generic early warning signals that do not require detailed knowledge of the system dynamics. These are typically obtained from time series by estimating a statistical indicator in a sliding window with appropriate detrending (see Sect. S4). For bifurcation tipping, a system often exhibits critical slowing down, which can be measured by increasing variance and autocorrelation. In Fig. 12



Other recent work II

"Dynamic tipping in the non-smooth Stommel-box model, with fast oscillatory forcing"

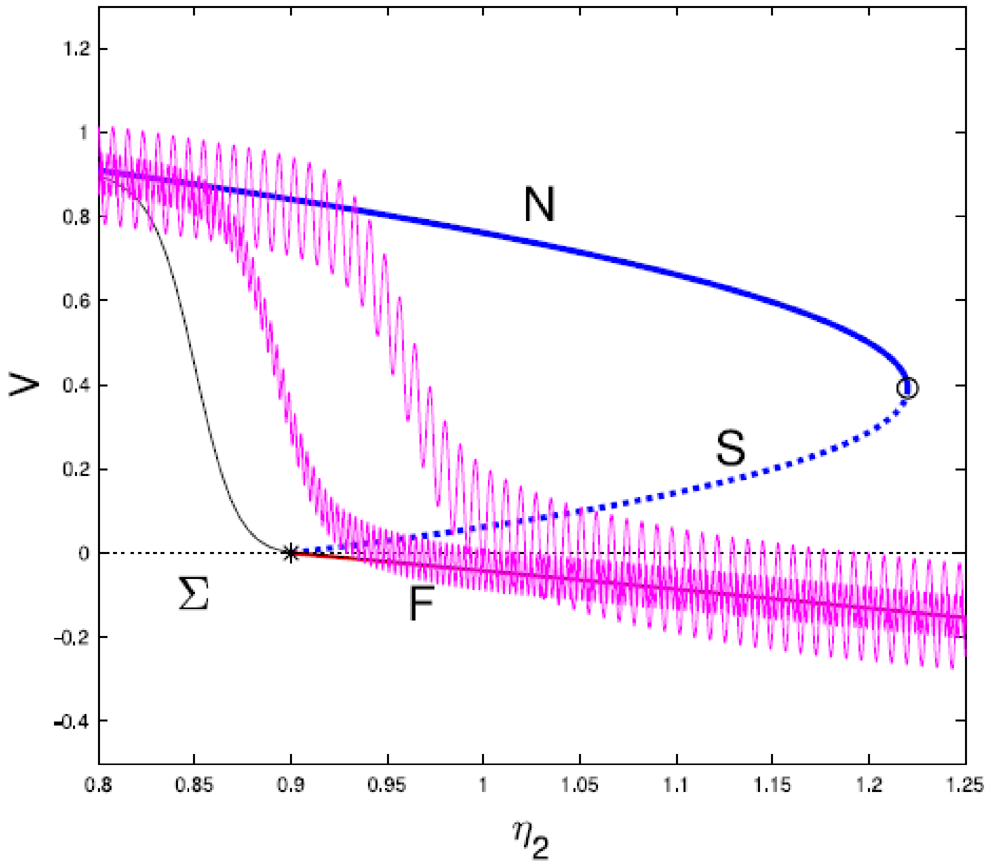
Chris Budd, Cody Griffith, Rachel Kuske

Physica D, April 30, 2021

$$\dot{\mathcal{V}} = \eta_1 - \eta_2(t) + \eta_3(\mathcal{T} - \mathcal{V}) - \mathcal{T} - \mathcal{V}|\mathcal{V}| + A \sin \Omega t$$

$$\dot{\mathcal{T}} = \eta_1 - \mathcal{T}(1 + |\mathcal{V}|) + B \sin \Omega t$$

$$\dot{\eta}_2 = -\epsilon, \qquad \epsilon \ll 1, \qquad \Omega \gg 1\,.$$



In the news (August 2021)

- ***NY Times (08/06/2021): "A Crucial System of Ocean Currents Is Faltering, Research Suggests"***

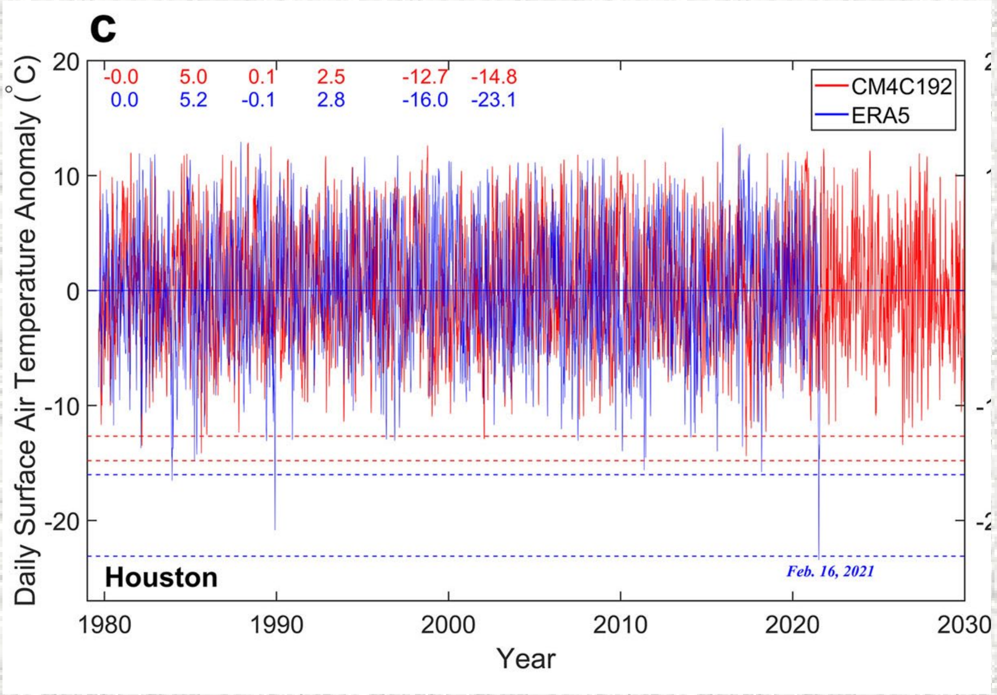
Observation-based early-warning signals for a collapse of the Atlantic Meridional Overturning Circulation. **Niklas Boers, Nature Climate Change (2021)**

"**…**Observations and recently suggested fingerprints of AMOC variability indicate a gradual weakening during the last decades, but estimates of the critical transition point remain uncertain…"

- ***Washington Post (08/02021): "A critical ocean system may be heading for collapse due to climate change, study finds" The consequences of a collapse would likely be far-reaching***

When Texas froze. Weakening of AMOC

From: “Influence of the Atlantic meridional overturning circulation on the U.S. extreme cold weather”, Jianjun Yin and Ming Zhao,
COMMUNICATIONS EARTH & ENVIRONMENT | (2021) 2:218 | https://doi.org/10.1038/s43247-021-00290-9 | www.nature.com/commsenv October 2021.



Are we at fault ?
From IPCC21 report

- In summary, models do not support robust assessment of the role of anthropogenic forcing in the observed AMOC weakening between the mid-2000s and the mid-2010s, which is assessed to have occurred with high confidence in Section 2.3.3.4.1
- Thus, we have low confidence that anthropogenic forcing has had a significant influence on 24 changes in AMOC strength during the 1860-2014 period

From SIAM

Check SIAM News, October 2021

Society for Industrial and Applied Mathematics

Research and Education Priorities to Address Climate Change, Boost Environmental Resilience, and Advance Clean Energy

Relatively simple Climate Models + Data can go a long way

The power of fluctuations, Nature Editorial, November 8, 2021

“**There is a deep humility that lies at the root of the scientific approach espoused by scientists such as Manabe, Hasselmann and Parisi**, which requires one to embrace uncertainty, variation and even doubt. Contrary to the perception that more is better, in order to understand a phenomenon it isn’t necessary to construct a faithful representation of the entire physical system including as many of the fine details as possible. But it is essential to identify the fundamental ingredients that really matter to understand the problem, and appreciate their (at times, unintended) effects over vastly different time scales.
When it comes to climate change, we must hope that we also find the collective humility to do something about it”.

Beyond the incredible intellectual challenge, we “owe it” to the younger generations

Thanks !

Seeking contributions, look for an announcement:
Special Issue on Climate Change (2022) in "Advances in Continuous and Discrete Models: Theory and Modern Applications"