NODAL CONTROL AND THE TURNPIKE PHENOMENON

Martin Gugat — Chair in Dynamics, Control, and Numerics (AvH Professorship), FAU Erlangen-Nürnberg

Rüdiger Schultz — Research Group Optimization and Discrete Mathematics, Universität Duisburg-Essen

Michael Schuster — Chair in Dynamics, Control, and Numerics (AvH Professorship), FAU Erlangen-Nürnberg

TRR 154 - Mathematical Modelling, Simulation and Optimization using the Example of Gas Networks (Subproject C03)

TRR 154
The "turnaround in energy policy" is currently in the main focus of public opinion. It concerns social, political and scientific aspects as the dependence on a reliable, efficient and affordable energy supply becomes increasingly dominant. On the other side, the desire for a clean, environmentally consistent and climate-friendly energy production is stronger than ever.

Natural gas, hydrogen and other energy sources:
To balance these tendencies while making a transition to nuclear-free energy supply, natural gas is an important energy source in the current decade. Hydrogen as energy source will play an important role in the near future. Hydrogen is an energy carrier and can deliver or store a tremendous amount of energy. Gaseous hydrogen can be transported through pipelines much the way natural gas is today. The mathematical results in this context are also applicable in other context, e.g., to physical transport systems like water networks.

Summary of Research Results (2018 - 2022)

Analytical solutions

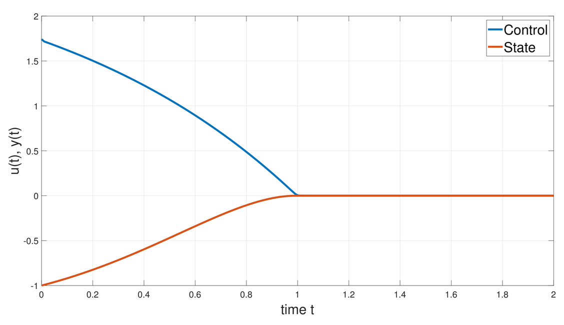
- Analytical solutions for **ISO2** and **ISO4** have been studied in detail. A multi-period optimization approach of steady states does not always converge to the optimal state [1].

Feedback stabilization

- Local exponential stability for the doubly nonlinear parabolic PDE **ISO3** with a boundary feedback flow control scheme [2]

Turnpike phenomenon for hyperbolic systems

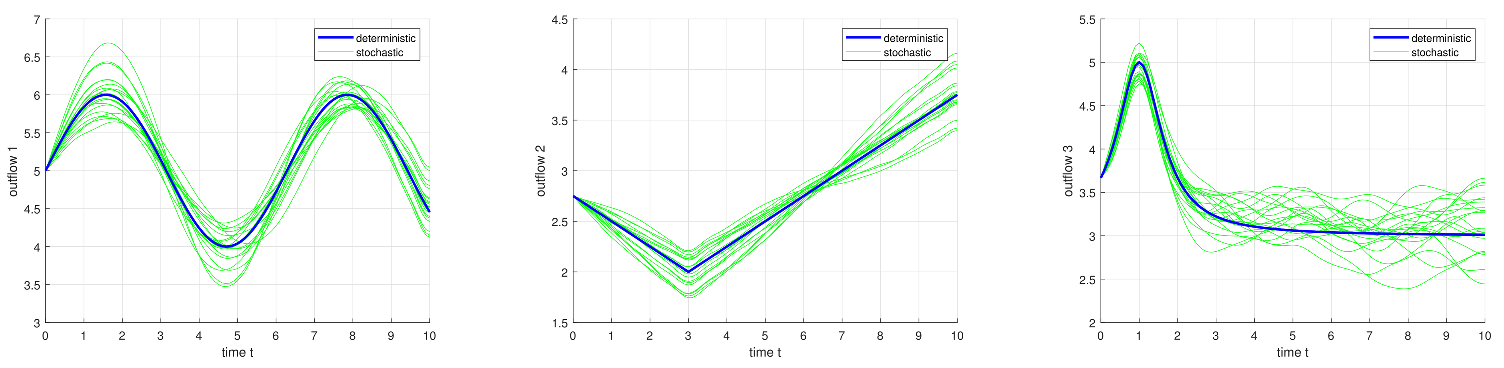
- Turnpike result for linear hyperbolic systems with convex objective function [3]
- Turnpike result for linear hyperbolic PDEs with and without integer constraint for sufficiently large time horizon [4]
- Finite-Time Turnpike result for optimal control problems: Control and state reach the stationary state exactly after finite time. With non-smooth tracking term in [5] and with smooth objective function in [6]



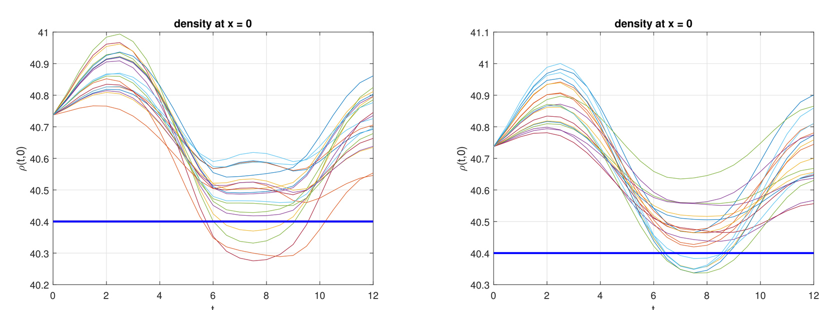
- Turnpike structure in the interior of the time interval $[0, T]$ for optimal control problems with pointwise-in-time constraints [7]

Optimization with probabilistic constraints

- Approximation of optimization problems with (dynamic) probabilistic constraints using a kernel density estimator approach [8] and application to gas networks [9]



- Existence of optimal Neumann boundary controls thet steer a system with uncertain initial state in a neighbourhood of a desired terminal state with high probability [10]
- Include H^2-norm of the control as control cost to the objective function of an optimal control problem with random gas demand to avoid large pressure fluctuations [11]



Market Models

- Coupling of the PDE model for the flow with a 4 level entry-exit market model from economics (network capacities, bookings, nominations, optimal transport plan): Numerical results show a piecewise turnpike property

Mathematical Models for the Gas Flow in Pipelines

ISO1
$$\begin{cases} \rho_t + (\rho v)_x = 0 \\ (\rho v)_t + \left(p + \rho v^2\right)_x = -\frac{\lambda}{2D}\rho v|v| - g\rho\sin(\alpha) \end{cases}$$

ISO3
$$\begin{cases} \rho_t + (\rho v)_x = 0 \\ p_x = -\frac{\lambda}{2D}\rho v|v| - g\rho\sin(\alpha) \end{cases}$$

ISO2
$$\begin{cases} \rho_t + (\rho v)_x = 0 \\ (\rho v)_t + p_x = -\frac{\lambda}{2D}\rho v|v| - g\rho\sin(\alpha) \end{cases}$$

ISO4
$$\begin{cases} (\rho v)_x = 0 \\ p_x = -\frac{\lambda}{2D}\rho v|v| - g\rho\sin(\alpha) \end{cases}$$

p	gas pressure	v	gas velocity	g	gravitational constant
ρ	gas density	λ/D	pipe friction	α	pipe slope

Further Goals and Mathematical Challenges

- Combination of methods from PDE constrained optimization, mixed integer programming and uncertainty
- The class of optimal boundary control problems with hyperbolic systems and probabilistic constraints has not been studied yet
- What type of turnpike result occurs in a probabilistic setting or with switching decisions?
- The topological structure of the graph leads to demanding tasks

Project Plan (2022 - 2026)

WP1: Existence of solutions and necessary optimality conditions for optimal boundary control problems for hyperbolic systems with dynamic probabilistic constraints

- Probabilistic constraints as modelling tool to obtain controls that are robust against fluctuations in gas demand

$$\mathbb{P}(\ g(u,\xi,\tau) \leq 0 \quad \forall \tau \in \mathcal{T}\ ) \ \geq \ p$$

WP2: The turnpike phenomenon for optimal control problems with probabilistic constraints

- Investigate the turnpike phenomenon for optimal boundary control problems with dynamic probabilistic constraints based upon the result of WP1

WP3: The turnpike phenomenon for problems with switching decisions and the finite-time turnpike property

- Does a turnpike structure exist for small switching cost?
- Does a turnpike property with interior decay occur?
- Does a finite-time turnpike phenomenon occur if the control is only allowed to attain values from a finite set?

WP4: Algorithms for stationary nonlinear gas flow on networks with interwinded cycles

- Transformation to a system of polynomials in the flow variables, tackle this system by symbolic computation ((comprehensive) Gröbner bases)

Selected publications

[1] Gugat, M., Krug, R., Martin, A. (2021). **Transient gas pipeline flow: Analytic examples, numerical simulation and a comparision to the quasi-static approach.** Optim. Eng.

[2] Gugat, M., Hante, F., Jin, L. (2020). **Closed loop control of gas flow in a pipe: Stability for a transient model.** at-Automatisierungstechnik.

[3] Gugat, M. (2019). **A turnpike result for convex hyperbolic optimal boundary control problems.** Pure Appl. Funct. Anal. 4(4), 849–866.

[4] Gugat, M., Hante, F. (2019). **On the turnpike phenomenon for optimal boundary control problems with hyperbolic systems.** SIAM J. Control Optim. 57(1), 264–289.

[5] Gugat, M., Schuster, M., Zuazua, E. (2021). **The Finite-Time Turnpike Phenomenon for Optimal Control Problems: Stabilizytion by Non-Smooth Tracking Terms.** Stabilization of Distributed Parameter Systems: Design Methods and Applications.

[6] Gugat, M. (2021). **Optimal boundary control of the wave equation: The finite-time turnpike phenomenon.** To appear in Math. Rep. (Bucur.).

[7] Gugat, M. (2021). **On the turnpike property with interior decay for optimal control problems.** Math. Control Signals Systems 33, 237–258.

[8] Schuster, M., Strauch, E., Gugat, M., Lang, J. (2022). **Probabilistic Constrained Optimization on Flow Networks.** Optim. Eng. 23, 1–50.

[9] Schuster, M. (2021). **Nodal Control and Probabilistic Constrained Optimization using the Example of Gas Networks.** Dissertation, FAU Erlangen-Nürnberg, Germany, available under `https://opus4.kobv.de/opus4-trr154/home`

[10] Farshbaf-Shaker M. H., Gugat, M., Heitsch, H., Henrion, H. (2020). **Optimal Neumann Boundary Control of a Vibrating String with Uncertain Initial Data and Probabilistic Terminal Constraints.** SIAM J. Control Optim. 58(4), 2288–2311.

[11] Gugat, M., Schuster, M. (2022). **Max-p optimal boundary control of gas flow.** To appear in MTNS Bayreuth.

[12] Gugat, M., Schuster, M., Steffensen, S. (2022). **A Dynamic Multilevel Model of the European Gas Market.** Submitted, preprint available under `https://opus4.kobv.de/opus4-trr154/home`