Friedrich-Alexander-Universität
Naturwissenschaftliche Fakultät

Friedrich-Alexander-Universität
DYNAMICS, CONTROL, MACHINE LEARNING AND NUMERICS

TWO EXAMPLES [2]

Christopher Sorg Chair in Dynamics, Control, and Numerics – AvH Professorship
FAU Erlangen-Nürnberg

Two Dimensions

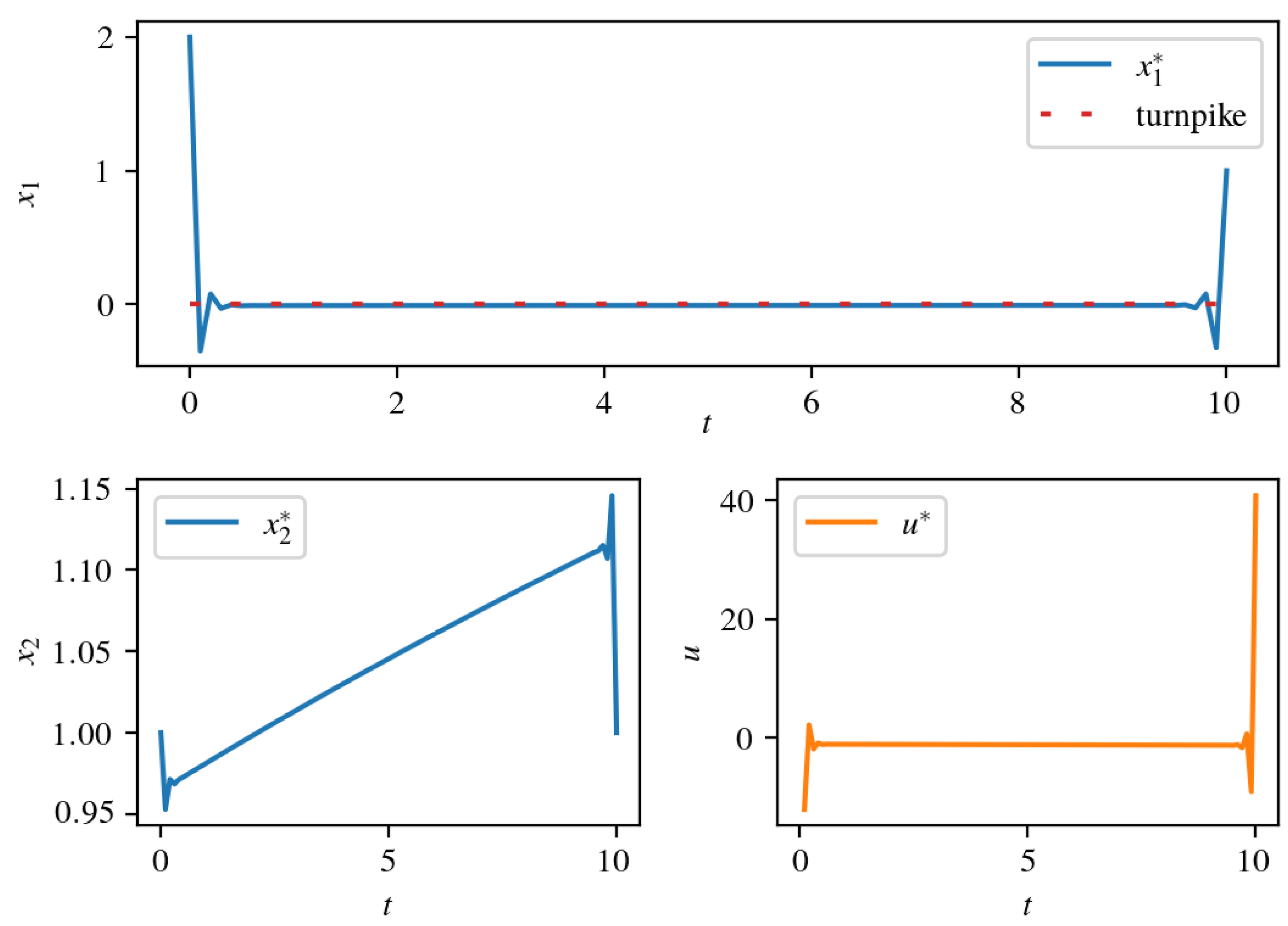
Consider the functions E, J, R, η and B defined by

$$E(x) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad J(x) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad B(x) = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$R(x) = \begin{bmatrix} \frac{1}{4}(4\|x\|^2+1)^2 & 0 \\ 0 & 0 \end{bmatrix}, \quad \eta(x) = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} x$$

for all $x \in \mathbb{R}^2$, which together with the Hamiltonian

$$\mathcal{H}(x) = \frac{1}{2}\, x^T \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} x$$

form a port-Hamiltonian system. Taking $x_0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $x_T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ for $T = 10$ and $-50 \leq u(t) \leq 50$ we got



This is consisent with the theory: For

$$f : \mathbb{R}^2 \to \mathbb{R}^2,\ x \mapsto R(x)^{1/2}\eta(x) = \begin{pmatrix} 4\|x\|^2+1 & 0 \\ 0 & 0 \end{pmatrix} x = \begin{bmatrix} 4(x_1^3 + x_2^2 x_1) + x_1 \\ 0 \end{bmatrix}$$

one has

$$\mathcal{M} = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 \mid x_1 = 0 \right\}.$$

Martin Gugat Chair in Dynamics, Control, and Numerics – AvH Professorship
FAU Erlangen-Nürnberg

Three Dimensions

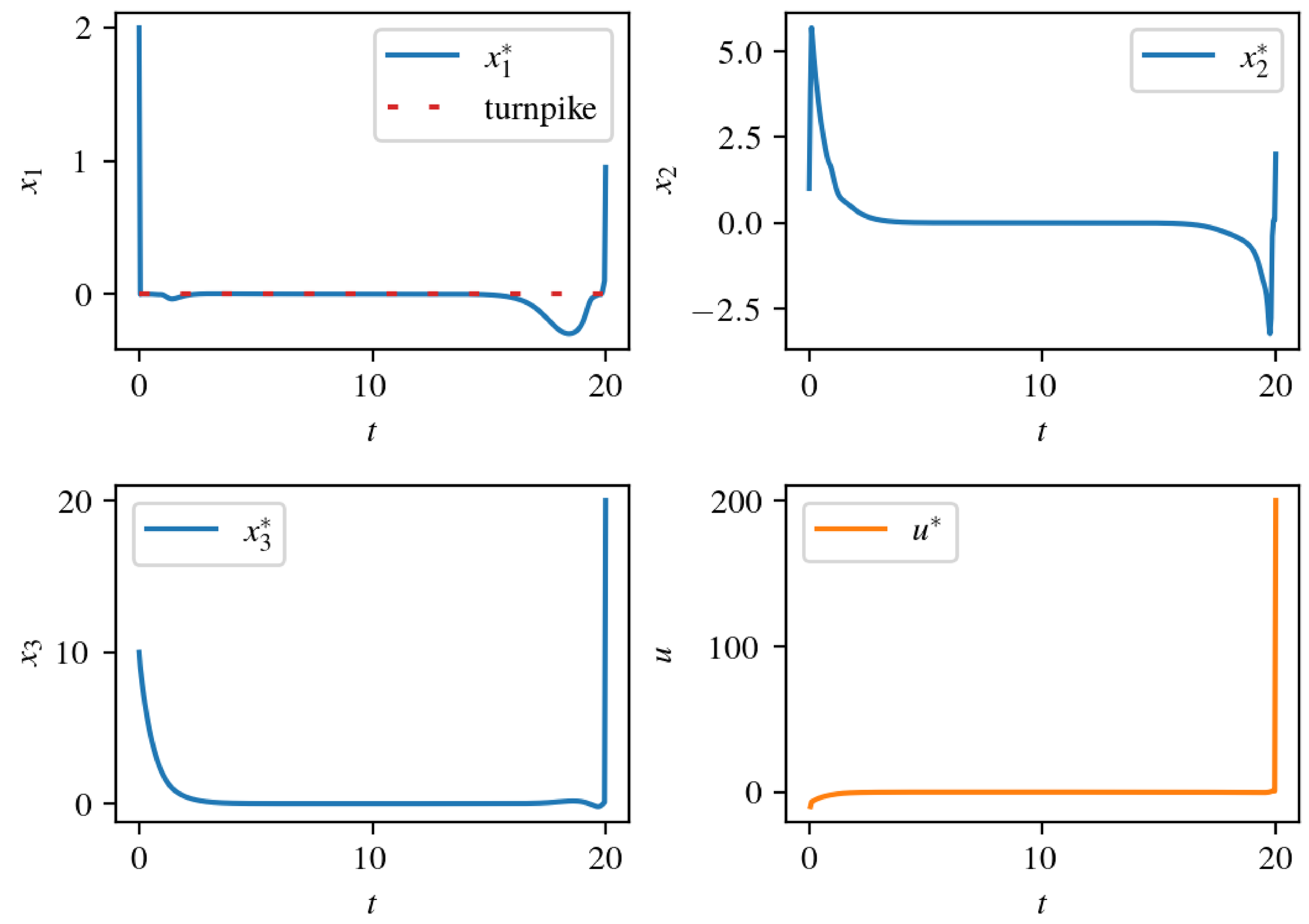
Consider the functions E, J, R, η and B defined by

$$E(x) := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad J(x) := \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B(x) := \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix},$$
$$R(x) := \begin{bmatrix} \frac{1}{4}(4x_1^2+4x_2^2+1)^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \eta(x) := \begin{bmatrix} 2x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

for all $x \in \mathbb{R}^3$, which together with the Hamiltonian

$$\mathcal{H}(x) = \frac{1}{2}\, x^T \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x$$

form again a port-Hamiltonian system. Taking $x_0 = \begin{bmatrix} 2 \\ 1 \\ 10 \end{bmatrix}$ and $x_T = \begin{bmatrix} 1 \\ 2 \\ 20 \end{bmatrix}$ for $T = 20$ and $-200 \leq u(t) \leq 200$ we got



This is consisent with the theory as one has

$$\mathcal{M} = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \mid x_1 = 0 \right\}.$$

Department MATHEMATIK

Unterstützt von / Supported by
Alexander von Humboldt Stiftung/Foundation