Abstract

Oberwolfach Workshop:

Polynomial Optimization for Nonlinear Dynamics: Theory, Algorithms, and Applications

Dates:

28 July - 2 August 2024 (Code: 2431)

Organizers:

Giovanni Fantuzzi, London
David Goluskin, Victoria
Jean-Bernard Lasserre, Toulouse

Polynomial optimization provides powerful tools to study nonlinear dynamics. The connection between these two fields arises because many questions about dynamical systems governed by nonlinear ODEs or PDEs can be formulated as optimization problems over auxiliary functions defined on the state space of the dynamics. A familiar type of auxiliary function is a Lyapunov function, which satisfies inequality constraints that imply nonlinear stability of an equilibrium state. Auxiliary functions satisfying different inequality constraints can imply bounds on average or extreme behavior, localize attractors, approximate basins of attraction, construct optimal control laws, and more. When all expressions are polynomial, inequality constraints can be enforced by sum-of-squares conditions, resulting in polynomial optimization problems that are computationally tractable due to recent advances. This approach is very successful for ODEs of low-to-moderate dimension, generally giving stronger results than any other method. Applications to PDEs and higher-dimensional ODEs are often impeded by computational cost and numerical ill-conditioning. These impediments have several causes and call for theoretical and algorithmic advances. To help surmount these challenges, this workshop will be the first to bring together experts from the different areas of mathematics that are needed: nonlinear dynamics, convex optimization, semialgebraic geometry, control theory, and variational analysis.