Historical introduction

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Outline

1. Historical introduction
   - Origins
   - Nowadays
   - Key concepts
   - Origins II
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Outline

MATHEMATICAL CONTROL THEORY,

or

CONTROL ENGINEERING

or simply

CONTROL THEORY?

An interdisciplinary field of research in between Mathematics and Engineering with strong connections with Scientific Computing, Technology, Communications,...

The origins

*"... if every instrument could accomplish its own work, obeying or anticipating the will of others ...
if the shuttle weaved and the pick touched the lyre without a hand to guide them, chief workmen would not need servants, nor masters slaves."*

Chapter 3, Book 1, of the monograph "Politics" by Aristotle (384-322 B. C.).

Main motivation: The need of automatizing processes to let the human being gain in liberty, freedom, and quality of life.

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- The state equation

$$A(y) = f(v). \tag{1}$$

- y is the state to be controlled.
- v is the control. It belongs to the set of admissible controls .
- Roughly speaking the goal is to drive the state y close to a desired state y_d:

$$y \sim y_d.$$

In this general functional setting many different mathematical models feet:

- Linear or nonlinear problems;
- Deterministic or stochastic models;
- Finite dimensional or infinite dimensional models;
- Ordinary Differential Equations (ODE) orPartial Differential Equations (PDE).

And, of course, when facing complex real life processes, often, hybrid models might also be needed.

Several kinds of different control problems may also feet in this frame depending on how the control objective is formulated:

- Optimal control (related with the Calculus of Variations)

$$min_{v \in Uad} ||y - y_d||^2.$$

- Controllability: Drive exactly the state y to the prescribed one y_d.
  This is a more dynamical notion.
  Several relaxed versions also arise: approximate controllability.
- Stabilization or feedback control. (real time control)

$$v = F(y); \quad A(y) = f(F(y)).$$

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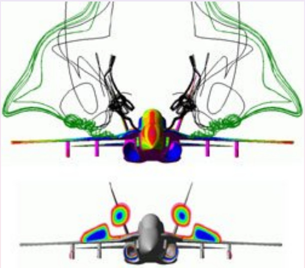
The concept of feedback. Inspired in the capacity of biological systems to self-regulate their activities.
Incorporated to Control Engineering in the twenties by the engineers of the “Bell Telephone Laboratory” but, at that time, it was already recognized and consolidated in other areas, such as Political Economics.

Feedback process: the one in which the state of the system determines the way the control has to be exerted in real time
Nowadays, feedback processes are ubiquitous in applications to Engineering, Economy also in Biology, Psychology, etc.

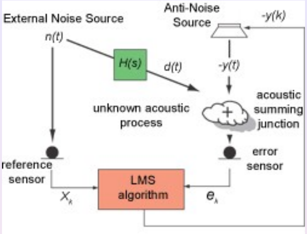
Cause-effect principle $\rightarrow$ Cause-effect-cause principle.

Some examples

- The thermostat;
- The control of aircrafts in flight or vehicles in motion:



- Noise reduction:



Noise reduction is a subject to research in many different fields. Depending on the environment, the application, the source signals, the noise, and so on, the solutions look very different. Here we consider noise reduction for audio signals, especially speech signals, and concentrate on common acoustic environments such an office room or inside a car. The goal of the noise reduction is to reduce the noise level without distorting the speech, thus reduce the stress on the listener and - ideally - increase intelligibility.

The need of fluctuations.

> *"It is a curious fact that, while political economists recognize that for the proper action of the law of supply and demand there must be fluctuations, it has not generally been recognized by mechanicians in this matter of the steam engine governor. The aim of the mechanical engineers, as is that of the political economist, should be not to do away with these fluctuations all together (for then he does away with the principles of self-regulation), but to diminish them as much as possible, still leaving them large enough to have sufficient regulating power."*

H.R. Hall, *Governors and Governing Mechanisms*, The Technical Publishing Co., 2nd ed., Manchester 1907.

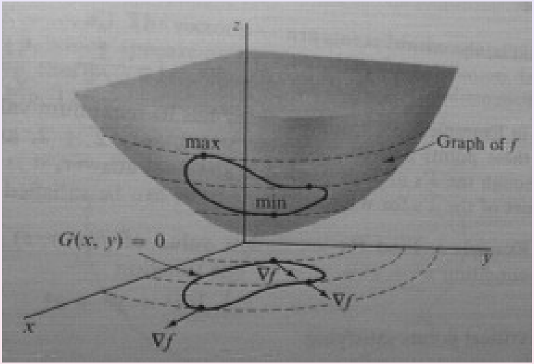
An example: Lagrange multipliers.

$$\min_{g(x)=c} f(x).$$

The answer: critical points x are those for which

$$\nabla f(x) = \lambda \nabla g(x)$$

for some real λ.
This is so because $\nabla g(x)$ is the normal to the level set in which minimization occurs. A necessary condition for the point x to be critical is that $\nabla f(x)$ points in this normal direction. Otherwise, if $\nabla(x)$ had a nontrivial projection over the level set $g(x) = c$ there would necessarilly exist a better choice of x for which $f(x)$ would be even smaller.

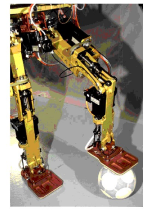

z
max
Graph of f
min
G(x, y) = 0
∇f
∇f
∇f
x
y

Cybernetics

"Cybernétique" was proposed by the French physicist A.-M. Ampère in the XIX Century to design the nonexistent science of process controlling. This was quickly forgotten until 1948, when N. Wiener (1894–1964) chose "Cybernetics" as the title of his book.

Wiener defined Cybernetics as " the science of control and communication in animals and machines".

In this way, he established the connection between Control Theory and Physiology and anticipated that, in a desirable future, engines would obey and imitate human beings.



In mathematical terms this corresponds to duality in convex analysis.
To each optimization problem it corresponds a dual one. Solving the primal one is equivalent to solving the dual one, and viceversa. But often in practice one is much easier to solve than the other one.
This duality principle is to be used to always solve the easy one.

PRIMAL =DUAL

CONTROL= COMMUNICATION

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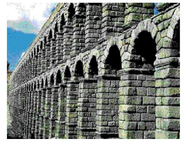
- Irrigation systems, ancient Mesopotamia, 2000 BC.
- Harpenodaptai, ancient Egypt, the string stretchers.
    - Primal: The minimal distance between two points is given by the straight line.
    - Dual: The maximal distance between the extremes of a cord is obtained when the cord is along a straight line.

In mathematical terms, things are not easy:
To minimize the functional

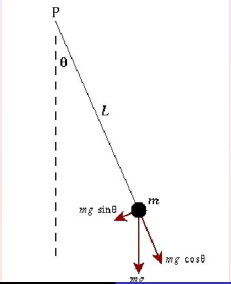
$$\int_0^1 ||x'(t)||dt$$

among the set of parametrized curves $x : [0,1] \to \mathbf{R}^d$, such that $x(0) = A$ and $x(1) = B$.
We easily end up working in the BV class of functions of bounded variation, out of the most natural and simple context of Hilbert spaces.

Roman aqueducts. Systems of water transportation endowed with valves and regulators.



The pendulum. The works of Ch. Huygens and R. Hooke, in the end of the XVII century, the goal being measuring in a precise way location and time, so precious in navigation.



The first mathematical rigorous analysis of the stability properties of the steam engine was done by Lord J. C. Maxwell, in 1868. The explanation of some erratic behaviors was explained. Until them it was not well understood why apparently more ellaborated and perfect regulators could have a bad behavior.
The reason is now refered to as the overdamping phenomenon.
Consider the equation of the pendulum:

$$x'' + x = 0.$$

This describes a pure conservative dynamics: the energy

$$e(t) = \frac{1}{2}[x^2(t) + |x'(t)|^2]$$

is constant in time.
Let us now consider the dynamics of the pendulum in presence of a friction term:

$$x'' + x = -kx',$$

k being a positive constant $k > 0$.

The energy decays exponently. But the decay rate does not necessarily increase with the damping parameter k.

Indeed, computed the eigenvalues of the characteristic equation one finds:

$$\lambda_{\pm} = [-k \pm \sqrt{k^2 - 4}]/2.$$

It is easy to see that λ_+ increases as $k > 2$ increases.

Indeed,

$$\lambda_+ = \frac{-k^2 + k^2 - 4}{k + \sqrt{k^2 - 4}} = \frac{-4}{k + \sqrt{k^2 - 4}}.$$

This shows that $\lambda_+ \to 0$ when $k \to +\infty$, so, the decay of the system gets worse when k is very large.

This confirms the prediction that optimal controls and strategies are often complex and that they do not necessarily obey to the very first intuition.

Automatic control. The number of applications rapidly increased in the thirties covering different areas like amplifiers in telecommunications, distribution systems in electrical plants, stabilization of aeroplanes, electrical mechanisms in paper production, petroleum and steel industry,...

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By that time there were two clear and distinct approaches:

- State space approach, based on modelling by means of Ordinary Differential equations (ODE);
- The frequency domain approach, based in the Fourier representation of signals.

$$\text{PHYSICAL SPACE} \equiv \text{FREQUENCY SPACE}$$

But after the second world war it was discovered that most physical systems were nonlinear and nondeterministic.

IMPORTANT CONTRIBUTIONS WERE MADE IN THE 60's:

- Kalman and his theory of filtering and algebraic approach to the control of systems;
- Pontryagin and his maximum principle: A generalization of Lagrange multipliers.
- Bellman and his principle of dynamic programming:
  A trajectory is optimal if it is optimal at every time.

IMPORTANT FURTHER DEVELOPMENTS HAVE BEEN DONE IN THE LAST DECADES CONCERNING:

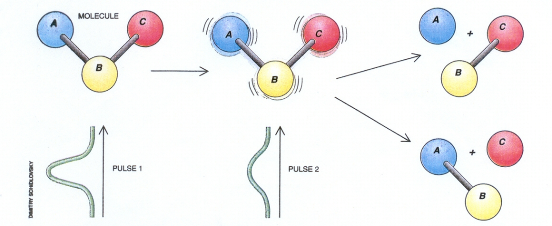
- Nonlinear problems;
  Lie brackets: Think on how park or unpark your car...
- Stochastic models;
  Human beings introduce more uncertainty in already uncertain systems...
- Infinite dimensional systems $=$ Partial Differential Equations (PDE), also referd to as Distributed Parameter Systems.
  When the number of degrees of freedom is too large one is obliged to deal with models in Continuum Mechanics....

IS PDE CONTROL RELEVANT?

The answer is, definitely, YES.
Let us mention some examples in which the wave equation is involved in a way or another.

- Noise reduction in cavities and vehicles.
  Typically, the models involve the wave equation for the acoustic waves coupled with some other equations modelling the dynamics of the boundary structure, the action of actuators, possibly through smart mechanisms and materials.

- Quantum control and Computing.
  Laser control in Quantum mechanical and molecular systems to design coherent vibrational states.
  In this case the fundamental equation is the Schrödinger one. Most of the theory we shall develop here applies in this case too. The Schrödinger equation may be viewed as a wave equation with inifnite speed of propagation.



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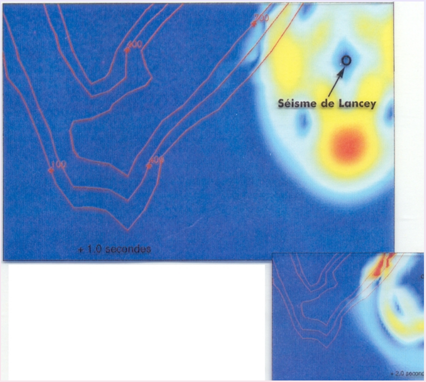
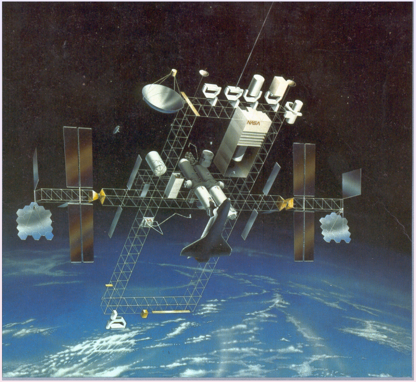
- Seismic waves, earthquakes.



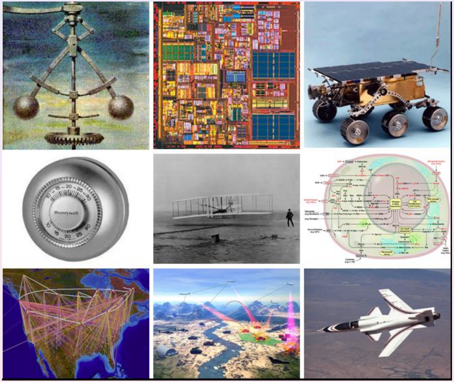
F. Cotton, P.-Y. Bard, C. Berge et D. Hatzfeld, Qu'est-ce qui fait vibrer Grenoble?, La Recherche, 320, Mai, 1999, 39-43.

- Flexible structures.



SIAM Report on "Future Directions in Control Theory. A Mathematical Perspective", W. H. Fleming et al., 1988.

An many others...



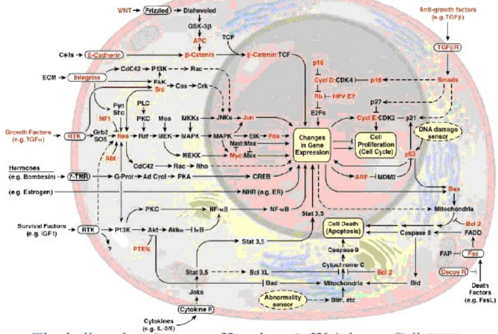
Control in an information rich World, SIAM, R. Murray Ed., 2003.

CONTROL THEORY is full of challenging, difficult and interesting mathematical problems.

Control is continuously enriched by the permanent interaction with applications.

This interaction works in both directions:

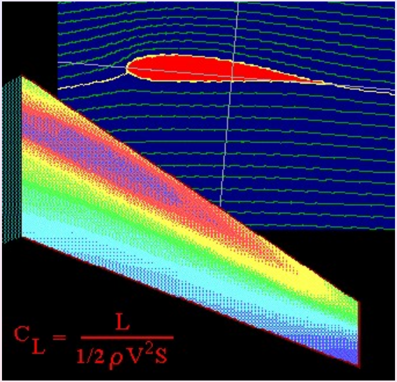
- mathematical control theory provides the understanding allowing to improve real-life control mechanisms;
- Applications provide and bring new mathematical problems of increasing complexity.



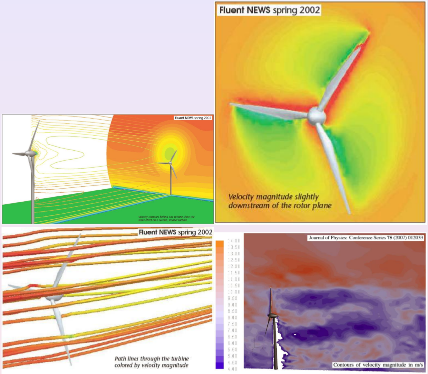
The hallmarks of cancer, Hanahan & Weinberg, Cell 2000

Biomedicine

Aerospace industry



Optimal shape design in aerodynamics



Eolic energy generation

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